# What is the harm in (partisan) gerrymandering? Collective vs. dyadic accounts of representational disparities

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## Abstract

Traditional approaches for documenting the harm of gerrymandering emphasize collective representation by legislatures, minimizing the relationship between individual voters and their respective representatives. Federal courts have struggled to map collective accounts onto cognizable constitutional harms, reflecting a discomfort evaluating a system of representation inescapably rooted in geographic districts using diagnostics that treat districts and their boundaries as an inconvenience rather than an intrinsic feature. A normative account of representation and accountability rooted in the dyadic relationship between voters and their legislators addresses the exact harms that courts have articulated yet struggled to substantiate. We derive a formal model of dyadic representation that yields a measure of disparities among different voters, including those divided by partisanship. We then compare enacted plans in four states against two million simulated counterfactuals, demonstrating how conclusions about the harms from gerrymandering may be highly sensitive to political factors such as polarization and officeholder motivation.

What, precisely, is the harm associated with partisan gerrymandering that might admit some form of judicial remedy? This deceptively simple question is surprisingly tricky to answer. In an immediate sense, no districting plan can be completely neutral, and in an electoral system with geographically defined districts, lines must be drawn somewhere. Moreover, any time communities are split into geographic districts, certain individuals and communities will fare worse than others. How, then, should we evaluate what counts as "worse," and how much disparity in outcomes ought we be willing to tolerate? State and federal courts have traditionally considered legal scholarship and political science as a means to unpack and diagnose the mischief of partisan gerrymandering, but have struggled to link this literature to the rights of individual voters to free and fair representation.

Much has been made of Chief Justice Roberts' quip during oral argument of a 2017 case that the metrics of partisan gerrymandering are "sociological gobbledygook."[1] However, the underlying concern that the Chief Justice was expressing was not the mathematical complexity of the metrics, but the lack of connection between these metrics and his understanding of individual rights as grounded in the Constitution. While pressing appellees to connect the dots between their preferred metric of gerrymandering—the Efficiency Gap (Stephanopoulos and McGhee 2015)—and a relevant constitutional harm, the Chief Justice said, "it is just not, it seems, a palatable answer to say the ruling was based on the fact that EG was greater than 7 percent. *That doesn't sound like language in the Constitution*," (*Id*. emphasis added). In the majority opinion that he would later author, Chief Justice Roberts reiterated this concern, writing that "the plaintiffs presented partisan-asymmetry studies showing that Act 43 had skewed Wisconsin's statewide map in favor of Republicans. Those studies do not address the effect that a gerrymander has on the votes of particular citizens. They measure instead the effect that a gerrymander has on the fortunes of political parties. That shortcoming confirms the fundamental problem with the plaintiffs' case as presented on this record. It is a case about group political interests, not individual legal rights" *Gill v Whitford*, 585 U.S. 48, 50 (2018).

To illustrate this "fundamental problem," consider an imaginary state composed of nine voters—five Republicans and four Democrats—that must be evenly divided among three districts. Two maps are proposed: A and B. Each column in the depiction below represents a district composed of three voters. In Map A, Republicans have a 2–1 majority in Districts 1 and 2 and Democrats have a 1–2 majority in District 3. In Map B, Republicans have a 3–0 majority in District 1 and a 2–1 majority in District 2, while Democrats enjoy a 0–3 majority in District 3.

| (1) | (2) | (3) | | (1) | (2) | (3) |
|-----|-----|-----|---|-----|-----|-----|
| R | R | R | | R | R | D |
| R | R | D | | R | R | D |
| D | D | D | | R | D | D |
| | Map A | | | | Map B | |

We begin with these two examples because they highlight the fundamental conceptual gulf between canonical approaches to the diagnosis of partisan gerrymanders on the one hand, and the jurisprudential approach taken by the federal courts before the Supreme Court held that partisan gerrymanders are nonjusticiable in federal courts in *Rucho v Common Cause* (2019).

What makes Map A different from Map B? From one perspective—grounded in theories of *collective* representation—all that matters is how closely the partisan breakdown of the elected assembly hews to that of the electorate as a whole. In that case, there is no meaningful difference between the maps: both have the same partisan breakdown in the electorate, and either would generate a delegation consisting of one Democratic and two Republican legislators.

From a different perspective—drawn from theories of *dyadic* representation—what matters is the agency relationship between voters in a given district and their representative. Viewed in this light, Maps A and B are not the same, and theories of dyadic representation provide insights into *why they differ*. Suppose voters only care about being represented by someone who reflects their values (McCartan and Kenny 2022). In Map A, four out of five Republicans and two out of four Democrats are represented by copartisans whereas in Map B, all five Republicans and 3/4 of Democrats are represented by copartisans. More voters are represented by a copartisan in Map B than Map A; moreover, Democrats make out nearly as well as Republicans on average in Map B. Consequently, we have two reasons—overall happiness and equal protection—to favor the second map over the first.

But now suppose that we have reason to be apprehensive of lopsided majorities. Perhaps the lack of competitiveness creates an environment where incumbents give less effort or shirk altogether. If that's the case, then Map B has a problem: specifically, six voters are in noncompetitive districts, compared to zero in Map A. If we care enough about this concern, then we will favor the first map over the second.

The comparison between Map A and Map B conveys several important lessons. First, there is a wedge between accounts of partisan gerrymandering grounded in theories of collective and dyadic representation: by construction, the two different maps perform identically on collective representation

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1   *Gill v Whitford*, 585 U.S. 684 (2017) (Transcript of Oral Argument at 38).

grounds.[2] From a dyadic perspective, the maps do not perform equally. But how we evaluate maps depends critically on the assumptions underlying our theory of dyadic representation. These assumptions pertain to different values, such as competitiveness, responsiveness, and representation at the district level. Depending on how we weigh these values, we might prefer one or the other map.

At the same time, however, there are some maps that perform poorly across both collective and dyadic theories of representation across an array of different assumptions. For example, consider Map C, which preserves the distribution of our nine voters but results in a majority of seats for Democrats (2/3) despite their minority position in the electorate.

| (1) | (2) | (3) |
|-----|-----|-----|
| R | R | R |
| D | D | R |
| D | D | R |

Map C

Districts 1 and 2 in Map C each have 1–2 Democratic majorities and District 3 has a 3–0 Republican majority. From both collective and dyadic representation perspectives, this map is terrible: (1) it denies a majority of the legislative delegation to the majority of voters, (2) it fails to maximize the number of voters represented by copartisans, and (3) it fails to minimize the number of lopsided districts.

Note that the conflict of values implicated in the foregoing examples only arises because voters are divided into districts. In a world where representatives are elected at-large on a party list, e.g. the geographic distribution of voters is irrelevant and all three maps would perform equally well on collective accounts because the elected assembly would be roughly proportionally representative of the electorate. Once individual districts are introduced, the quality of representation depends on a number of district-specific values and conditions such as competitiveness and polarization. A recognition of the potential tension between collective and dyadic theories of representation was brought to the fore in a recent op-ed by the authors of the Efficiency Gap who note that the US House of Representatives is "balanced" as of 2025 according to their collective account, but note that this balance is not the product of fair congressional plans in each state. "[T]o the contrary, many states have unfair plans, but Democratic and Republican gerrymanders happen to offset one another almost perfectly. The equipoise is fortuitous, but *it continues to leave many Americans poorly represented by their states' congressional delegations*" (Stephanopoulos, McGhee, and Warshaw 2025).

In this article, we articulate a theoretical account that draws a connection between partisan gerrymandering and dyadic representational harms to individual voters. Our inquiry proceeds in three parts. First, we review the federal and state jurisprudence on partisan gerrymandering and a number of quantitative measures commonly invoked either to attack or to defend specific districting plans. In the second part of the paper, we locate a fundamental issue plaguing the jurisprudence on partisan gerrymandering in the disconnect between the vocabulary legal and political science observers employ to discuss, diagnose, and evaluate partisan gerrymanders on the one hand and the constitutional and legal criteria employed by courts to establish harms to individuals on the other. That vocabulary, we demonstrate, is grounded in a collective account of representation. We demonstrate that a class of commonly employed gerrymandering diagnostics (those based on deviations from a prescribed seats-to-votes curve) may be reconceptualized as capturing deviations from a utilitarian ideal under the collective representation model. We also demonstrate that efforts to derive a measure of collective representational *disparities* as part of an equal protection claim must either focus on benefits and harms to parties rather than individuals (a move the Supreme Court has explicitly rejected);[3] rely on ad hoc and unjustifiable assumptions about what voters care about; or collapse into a *reductio ad absurdum*, namely,

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[2]  For example, both maps are observationally equivalent from the perspective of the Efficiency Gap. In each map, there are two wasted Democratic votes and one wasted Republican vote, yielding an Efficiency Gap of 1/9 in each.

[3]  To be sure, some Supreme Court Justices have argued that partisan gerrymandering implicates the constitutional rights of parties in addition to individual voters. *See Gill v Whitford*, 138 S.Ct. 1916, 1934–1938 (2017) (J. Kagan concurring) ("Partisan gerrymandering no doubt burdens individual votes, but it also causes other harms. And at some points in this litigation, the plaintiffs complained of a different injury—an infringement of their First Amendment right of association" … "Members of the disfavored party[,] deprived of their natural political strength[,] may face difficulties fundraising, registering voters, [and] eventually accomplishing their policy objectives." … "what is true for party members may be doubly true for party officials and triply true for the party itself."). *See also Vieth v Jubelirer*, 541 U.S. 267, 314 (2004) (J. Kennedy concurring) ("First Amendment concerns arise where a State enacts a law that has the purpose and effect of subjecting a group of voters or

a prescription that parties must have equal representation in a legislature or legislative delegation, irrespective of the distribution of partisanship in the electorate.

Due to the geographic nature of representation in the USA.[4] and the focus on individual rights in constitutional gerrymandering litigation, it is unsurprising that theories of collective representation have faltered as the wellspring of justiciable disparate impact claims. Thus, efforts to conceptualize, measure, and diagnose the benefits of representation (and the harm of underrepresentation) must necessarily ground their theory elsewhere.

We argue that a dyadic account of representation has sufficiently deep roots to serve as a foundation on which to develop this alternative. Per the dyadic account, representatives convene in a legislative assembly to faithfully represent the ideological, regional, and cultural characteristics of their districts, advocate for the economic and social well-being of their constituents, and work hard in potentially innumerable ways on their behalf. This conception is key to understanding the historical development of representation in the USA. For example, in his landmark book *The Ideological Origins of the American Revolution*, Bailyn (1967) describes the genesis of America's representative system, which he notes was "the first serious intellectual problem to come between England and the colonies." The British Parliament reflected a more collective notion of representation where members "came to sit not merely as parochial representatives, but as delegates of all the commons of the land" (*Id.*). However, because members of Parliament lacked a shared natural interest with the population in the colonies, the founders felt that their interests were not adequately accounted for and, as a result, adhered to a more dyadic conception. In the new USA, representatives were to be considered "creatures of their constituents" such that "people were present through their representatives, and were themselves, step by step and point by point, acting in the conduct of public affairs. No longer merely an ultimate check on government, they *were* in some sense the government. Government had no separate existence apart from them; it was *by* the people as well as *for* the people" (*Id.*).

Centering constituents as the unit of interest is merely the first step for developing a conceptually coherent understanding of the relationship between partisan gerrymandering and the promise of free and fair representation. The second step requires an answer to the question, *how* are these individuals harmed? Historically, the individual harm of partisan gerrymandering has been framed in terms of vote dilution, though scholars have long understood that vote dilution "does not fit easily within a conventional individual rights framework" (Gerken 2001).

We take a different approach, presenting a heuristic formal model intended to provide a coherent account of some important values underlying what it means for individuals to be represented well or poorly from the dyadic perspective: (1) the likelihood that they will be represented by someone who does or does not share their values; (2) the likelihood that they will be represented by someone who is more or less competent than an alternative; and (3) the likelihood that a voter will be represented by someone who is or isn't strongly motivated to work hard on their behalf. Of course, this list is not exhaustive. However, our simplified model clarifies how partisan gerrymandering is directly related to individual benefits and harms and yields an immediate measure of dyadic representational disparity (RD) between different groups of voters. This measure may be tailored to account for different assumptions about the political environment (including, e.g. partisan polarization and the attractiveness to an incumbent of remaining in office).

In the third part of the paper, we demonstrate how an analyst might employ our measure of dyadic RD in practice. Most importantly, we advocate two "best practices." First, rather than committing to a particular narrow set of assumptions about the environment in which redistricting happens, we recommend examining the *sensitivity* of our measure of RD to changes in those assumptions. Equal protection claims made against any particular plan will be strengthened to the extent that a map appears to strongly favor one group of voters over another across a broad range of assumptions.[5] Second, because irreducible geographic constraints might limit how "fair" a map can conceivably be for one or another group of voters, we recommend against articulating absolute standards against which to evaluate the measure. Instead, we endorse an outlier analysis approach in which the analyst generates

their party to disfavored treatment by reason of their views."); *Rucho v Common Cause*, 139 S.Ct. 2484, 2514 (2019) (*J.* Kagan dissenting) ("And partisan gerrymandering implicates the First Amendment too.").

   4   For example, 15 Stat. 450 (1842) (mandating single-member districts for congressional elections).

   5   To the extent that a specific set of assumptions may be grounded in a localized empirical assessment of voter preferences, candidate characteristics, and structural features of the political environment, the case will be strengthened still further.

an ensemble of counterfactual maps against which to compare specific proposed and enacted plans (Chikina, Frieze, and Pegden 2017; Fifield et al. 2020; DeFord, Duchin, and Solomon 2021). In the last part of this article, we show how different versions of the dyadic RD measure compare to traditional measures based on collective accounts (specifically, partisan bias and the Efficiency Gap) using case studies from Massachusetts, Florida, Pennsylvania, and New York.

To be sure, we are under no illusions that our argument will convince a majority of the Supreme Court that refocusing the study of gerrymandering on dyadic accounts of representation will generate sufficient constitutional grounds for revisiting *Rucho*. At the same time, partisan gerrymandering cases will continue to be litigated in state courts, and independent commissions are likely to draw on the best available metrics to ensure that their maps are fair. In this context, our enterprise provides a richer palette of normative foundations—and hence potential constitutional guideposts and remedies—for addressing the harms alleged in those cases.

## 1.  THE LAW OF GERRYMANDERING

Gerrymandering is widely considered to be harmful to the democratic process, *Rucho v Common Cause*, 588 U.S. 684, 718 (2019) (noting that excessive partisanship in districting is "incompatible with democratic principles"). In particular, gerrymandering has been blamed for increasing polarization and dysfunction in government and contributing to a disengaged electorate (Issacharoff and Pildes 1998; Issacharoff 2002). To limit abuse of the districting process, courts have struck down maps that deprive voters of "fair and equal representation."[6] The Supreme Court has struggled, however, in translating the underlying normative concern about fair and effective representation, with all of its technical complexity, into a manageable legal test. In other words, the question is not just whether political gerrymandering is a constitutional problem, but whether the federal courts should or even can address it. This turns on whether or not the Court can identify a judicially discernible and manageable standard to channel and limit judicial intervention (Charles and Fuentes-Rohwer 2018). Gerrymandering thus presents two distinct but related questions: justiciability (can there be a standard?) and identification (what is it?). These questions implicate both the relationship between electoral districting practices and constitutionally significant representative norms and the proper role of the federal judiciary in regulating electoral districting practices pursuant to these norms. Both are democratic problems of profound constitutional significance on which the Constitution provides limited explicit guidance.

### 1.1  Assessing Gerrymandering Claims in the Courts
#### 1.1.1  *Federal Courts: 1960s–2019*
In 2019, the US Supreme Court held that federal courts were ill-equipped to answer this question. Nevertheless, the question remains at the core of dozens of cases in state courts, and it continues to attract the attention of political actors, legal scholars, political theorists, mathematicians, and others. The US Supreme Court has acknowledged that some level of partisan gerrymandering is expected in America's political system.[7] Further, under the Voting Rights Act, jurisdictions are permitted to gerrymander districts as a *remedy* to the harm of political isolation in places with a history of social or state-sponsored discrimination, *Thornburg v Gingles*, 478 U.S. 30 (1986). How should districting plans

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[6]    *See, e.g. Reynolds v Sims*, 377 U.S. 533 (1964) ("Since the achieving of fair and effective representation for all citizens is concededly the basic aim of legislative apportionment, we conclude the Equal Protection Clause guarantees the opportunity for equal participation by all voters in the election of state legislators."); *Gaffney v Cummings*, 412 U.S. 735 (1973) (Referring to the "worthy goal" of "fair and effective representation"); *Evenwel v Abbott*, 136 S. Ct. 1120 (2016) ("By ensuring that each representative is subject to requests and suggestions from the same number of constituents, total-population apportionment promotes equitable and effective representation."); *Davis v Bandemer*, 478 U.S. 109 (1986) (J. Powell concurring) ("A standard that judges the constitutionality of a districting plan solely by reference to the doctrine of "one person one vote" may cause two detrimental results. First, as a perceived way to avoid litigation, legislative bodies may place undue emphasis on mathematical exactitude, subordinating or ignoring entirely other criteria that bear directly on the fairness of redistricting. Second, as this case illustrates, and as *Reynolds v Sims* anticipated, exclusive or primary reliance on "one person one vote" can betray the constitutional promise of fair and effective representation by enabling a legislature to engage intentionally in clearly discriminatory gerrymandering"); *Vieth v Jubelirer*, 541 U.S. 267 (2004) (J. Kennedy concurring) ("The object of districting is to establish 'fair and effective representation for all citizens.' *Reynolds v Sims*. At first it might seem that courts could determine, by the exercise of their own judgment, whether political classifications are related to this object or instead burden representational rights. The lack, however, of any agreed upon model of fair and effective representation makes this analysis difficult to pursue.")

[7]    *See Rucho v Common Cause*, 139 S.Ct. 2484, 2497 (2019) ("To hold that legislators cannot take partisan interests into account when drawing district lines would essentially countermand the Framers' decision to entrust districting to political entities."); and *Hunt v Cromartie*, 526 U.S. 541, 551 (1991) ("Our prior decisions have made clear that a jurisdiction may engage in constitutional political gerrymandering . . .").

be evaluated to assess the net harm to individuals in a given jurisdiction? One approach would simply evaluate the intent behind a particular districting plan: striking down plans that are intentionally designed to harm a subset of voters and upholding plans that are intentionally designed to help a subset of voters (Levitt 2018). Another approach would assess the harm to each individual or group in a particular plan against counterfactual plans and prioritize plans that minimize the social harm. The primary challenge for this second approach is that the harm of gerrymandering on individuals is highly undertheorized. "The central problem," as Justice Kennedy wrote in 2004, "is deciding when gerrymandering has gone too far," *Vieth v Jubelirer*, 541 U.S. 267, 296 (2004). In practice, courts have tied themselves into knots trying to determine how far is "too far." Although political gerrymanders undoubtedly implicate constitutional values, the Constitution's text offers limited procedural guidance on congressional and state legislative elections. And the precise scope of the federal judicial power is also a question without an explicit textual answer. In this sense, both action and inaction by the Court on political gerrymandering claims present real but ineffable constitutional risks.

The Guarantee Clause of the US Constitution, which assures every state a "republican form of government," has been historically invoked in challenges to partisan gerrymandering.[8] However, the US Supreme Court has consistently rejected Guarantee Clause arguments in this context. In cases like *Baker v Carr* (1962) and more recently in *Rucho v Common Cause* (2019), the Court has held that gerrymandering claims rooted in the Guarantee Clause present nonjusticiable political questions. This means that such claims are not suitable for judicial resolution because they lack judicially manageable standards. The Court's reluctance to engage with the Guarantee Clause stems from its historical context and the perceived need to maintain judicial neutrality in matters that are fundamentally political.[9]

The Equal Protection Clause of the Fourteenth Amendment has been a more fertile ground for challenging partisan gerrymandering. This Clause, which requires states to provide equal protection of the laws to all persons, has been interpreted to prohibit states from diluting the voting power of individuals based on their political affiliation. In *Davis v Bandemer*, 478 U.S. 109 (1986), the Supreme Court held that an equal protection claim could be raised to challenge a partisan gerrymander. However, the Court struggled to define a precise standard for when partisan gerrymandering violates the Equal Protection Clause. This ambiguity has persisted in more recent years. Consider the 2004 case *Vieth v Jubelirer*[10] where four Justices held that partisan gerrymandering is nonjusticiable due to the lack of judicially manageable standards. Four different Justices disagreed and argued that there were clear standards for determining when a partisan gerrymander had gone too far but could not agree among themselves what those standards should be, proposing three separate tests. More recently still, in *Gill v Whitford*, the Court underscored the necessity of proving concrete and individual harm to bring a successful claim under the Equal Protection Clause but avoided articulating any particular metric for proving this harm.[11]

Grasping for other constitutional hooks, litigants have raised First Amendment challenges to partisan gerrymandering. This approach frames partisan gerrymandering as a form of viewpoint discrimination, which is generally prohibited under the First Amendment. The argument posits that by diluting the electoral influence of certain voters based on their political beliefs or affiliations, the state effectively penalizes individuals for their protected speech and association. In cases like *Vieth v Jubelirer* (2004)

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8    *See, e.g.* Motion to Affirm of LWV of North Carolina, p. 5, *Rucho v Common Cause*, ("That partisan gerrymandering may offend the Guarantee Clause does not prevent it from breaching other constitutional provisions too."); Brief of Republican State Leadership Committee as Amici Curiae, p. 5, *Gill v Whitford*, 585 U.S. 48 (2019) ("Instead, this case should be viewed as raising, if anything, questions under the Guarantee Clause, which is the proper conceptual framework by which to view arguments that the electoral process is not sufficiently responsive to popular will."); Brief of Current Members of Congress and Bipartisan Former Members of Congress as Amici Curiae, p. 11, *Gill v Whitford*, 585 U.S. 48 (2019) ("The Constitution also conferred specific powers on the federal government to ensure the integrity of the system of government established by the Constitution. The Guarantee Clause empowers the United States to guarantee to every State in this Union a Republican Form of Government, U.S. Const. art. IV, §4, protecting the Constitution's system of government from aristocratic or monarchical innovation.") (internal quotations omitted); Brief for the ACLU and the Brennan Center for Justice as Amici Curiae, pp. 24–26, 541 U.S. 267 (2004) ("The federal government must 'guarantee to every State in [the] Union a Republican Form of Government … Partisan gerrymanders frustrate this command.'"), Brief of the Republican National Committee as Amici Curiae, pp. 14–15, n. 11, *Bandemer v Davis*, 603 F.Supp. 1479 (S.D. Ind) ("An alternative, though related, constitutional base could be Article IV §4 of the Constitution—The Guarantee Clause.")

9    *See, e.g. Vieth v Jubelirer*, 541 U.S. 267, 306–307 (2004) (J. Kennedy concurring) ([Excessive judicial regulation of electoral districting] "would commit federal and state courts to unprecedented intervention in the American political process," and without "rules to limit and confine judicial intervention … intervening courts—even when proceeding with the best intentions—would risk assuming political, not legal, responsibility for a process that often produces ill will and distrust").

10    541 U.S. 267 (2004).

11    585 U.S. 48 (2018).

and again in *Rucho v Common Cause* (which was consolidated with an explicit First Amendment challenge in *Lamone v Benisek*), plaintiffs have argued that partisan gerrymandering infringes on the First Amendment rights of free speech and association. However, the Supreme Court has refused to recognize partisan gerrymandering claims under the First Amendment, leading to ongoing legal debates and uncertainties.

### 1.1.2 State Courts: 2020–present

Although the Supreme Court held that *federal courts* are the improper venue for litigating partisan gerrymander claims, the majority noted that "[o]ur conclusion does not condone excessive partisan gerrymandering. Nor does our conclusion condemn complaints about districting to echo into a void. The States, for example, are actively addressing the issues on a number of fronts," *Rucho v Common Cause* at 719.

State courts have been relatively active in the years since *Rucho*, with partisan gerrymandering claims being raised against congressional plans in eleven states after the 2000 Census and redistricting cycle. A similar number of challenges have been raised against state legislative districts as well. As the Court noted in *Rucho*, there are a variety of legal arguments available to plaintiffs in state courts that are unavailable in federal courts (Wang, Ober and Williams 2019). First, every state constitution protects the right to free speech and expression (Koningisor 2024), and half of state constitutions contain their own Equal Protection Clause (Parness 2010). Some state courts have interpreted these clauses identically to the federal constitution, but other state courts provide stricter protections under their state constitutions despite nearly identical language (Douglas 2014). In addition, some states have adopted specific provisions that address the redistricting process. For example, the Florida constitution prohibits the drawing of districts "with the intent to favor or disfavor a political party or incumbent," Fla. Const. Art. III §20, and the Ohio constitution requires that the proportion of seats for a given party "shall correspond closely to the statewide preferences of the voters of Ohio," Ohio Const. Art. XI §6(b). As a result, different states rely on different languages when evaluating their state maps. For example, in 2022, the North Carolina Supreme Court struck down the state's congressional maps because they determined that the maps violated the "Free and Fair Elections" clause *and* the Equal Protection Clause of their state constitution, *Moore v Harper*, No. 413PA21 (February 14, 2022). In 2023, the Wisconsin Supreme Court struck down the state's legislative districts for failure to comply with the state constitutional requirement of contiguity, *Clarke v Wisc. Elec. Cmm'n*, No. 2023AP 1399-OA (December 22, 2023).

While state courts have been less stingy than their federal counterparts about the language in their respective constitutions, the underlying facts and metrics have remained the same. In a review of expert reports and orders by state courts since 2022, we find that plaintiffs continue to rely on the very same metrics—partisan asymmetry and the Efficiency Gap—that the Supreme Court dismissed as "sociological gobbledygook" and claimed were not amenable to evaluation by (federal) judges.[12] In other words, in the absence of an alternative, state courts continue to interpret the harm of gerrymandering in collective terms—does the state legislature as a whole reflect the statewide vote shares for each party?

As we show below, this approach suffers from two problems. First, collective representation "invariably sound[s] in a desire for proportional representation" even though the Court has written that "[o]ur cases, however, clearly foreclose any claim that the Constitution requires proportional representation or that legislatures in reapportioning must draw district lines to come as near as possible to allocating seats to the contending parties in proportion to what their anticipated statewide vote will be," *Rucho v Common Cause* at 704–705. Second, to the extent that we care as much about partisan *disparities* in the relationship between vote share and seat share as we do about the absolute number, a close examination of collective representation reveals that the collective account cannot inform our understanding of how voters are actually (disparately) harmed. This is because, under a theory of collective representation, voters are differentially harmed as legislative delegations move further away from 50–50 splits, *regardless of the underlying distribution of voters*.

While fairness to parties has been a central feature of redistricting cases, deviations from partisan fairness miscast the problem of gerrymandering. Looking back at the roots of this jurisprudence, deviation from fair and effective representation for individual voters is the core challenge to overcome.

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[12] In more than 1,200 mentions of gerrymandering metrics in these reports and opinions, 75% of citations by parties are metrics rooted in seats-to-votes curves as are 60% of citations by amici.

Accordingly, the jurisprudence of gerrymandering should center on individual voters and not on political parties. To do so, however, requires a theory of fair and effective representation.

## 1.2 Measuring Constitutional Culpability and Harm

### 1.2.1 *Fair and effective representation*

In particular, the Court has struggled to disentangle the harm of gerrymandering that accrues to groups like political parties and the harm of gerrymandering that accrues to individual voters. This tension was on full display in the Court's 2018 opinion in *Gill v Whitford*, when it dismissed a partisan gerrymandering case on the grounds that the individual plaintiffs lacked standing to assert what the Court conceptualized as a group harm. This focus on groups was rooted in First Amendment concerns about the associational rights of parties and also a sense that an individual voter is only harmed to the extent that her preferred party lacks power commensurate to its share of *all* votes.

By failing to delineate group and individual harms, the Court has created a Gordian knot: On the one hand, plaintiffs must show that they have been individually harmed in a way that gives them standing to sue. Likewise, they must present evidence to satisfy the Court's constitutional inquiry, which is rooted in a reading of the Equal Protection Clause that traditionally implicates harms to individuals. On the other hand, the evidence that they have been asked to present defines harm at the group level.

Much of the quantitative academic literature on gerrymandering has attempted to resolve the group/individual dilemma by relying on measures such as partisan symmetry and the Efficiency Gap, which are based on individual vote choices yet center the political welfare of parties. However, there is a major disconnect between the problem being addressed—lack of "fair and effective representation"—and these proxies that have been used to measure deviations from this ideal. Fairness to the parties has become central in part because it's easy to diagnose/measure, but "fair representation" (the Court's central concern since at least the 1960s) is not rooted in partisan fairness but instead in a concern about the consequences to voters of effective representation.

### 1.2.2 *Modern gerrymandering metrics*

To better understand the various metrics used to diagnose partisan gerrymandering, we delineate them into two distinct categories. The first category consists of measures grounded in the "seats-to-votes curve." Suppose there are two parties, $A$ and $B$, and let $v$ be the share of the electorate composed of members of party $A$ (and $1-v$ the corresponding share composed of members of party $B$). Let $s$ be the share of the legislature (or state delegation to Congress) composed of representatives from party $A$ (and $1-s$ the corresponding share from party $B$). A *seat-to-vote* curve is a one-to-one correspondence between the values of $v$ and $s$. Pure proportionality is achieved with a seat-to-vote curve of $s = v$, i.e. when the seat share precisely equals the vote share (up to the rounding error that invariably arises given the smaller number of legislators).

However, given the practical difficulty of achieving proportionality in first-past-the-post, single-member district electoral systems (Barton 2022), and the Supreme Court's repeated holding that proportionality is not mandated by the US Constitution, other scholars have proposed seat-to-vote curves that depart from proportionality as more practicable (and characterized by greater correspondence with empirical reality): examples include a cubic functional form (Tufte 1973) and quadratic (Barton 2022). The Efficiency Gap, about which we will have more to say below, yields a seat-to-vote curve of 0 for $v < 0.25$, 1 for $v > 0.75$, and $-0.5 + 2*v$ for values of $v$ between 0.25 and 0.75. Because of the slope of 2 for the intermediate range, the seat-to-vote curve for the Efficiency Gap is sometimes called "double-proportionality." Fig. 1 displays these seat-to-vote curves graphically.

A common approach for diagnosing a partisan gerrymander is to consider whether a given map may be expected to generate a (seats, votes) combination that deviates substantially from one of the prescribed curves. Of particular relevance is the principle of *partisan symmetry,* which states that if party $A$ receives a particular seat share for a given vote share, then party $B$ should receive the same seat share if it had received the same share of the electorate (King and Browning 1987; Grofman and King 2007). Political scientists have suggested methods to combine historical election data with hypothetical results based on hypothesized statewide swings to trace out an empirical seat-to-vote curve to compare with a postulated normative ideal. One method estimates the *partisan bias* that would exist if, in a counterfactual world in which the statewide vote tally was evenly split between the two major parties, the seat allocation would be similarly split. (Note that all of the proposed seat-to-vote curves pass
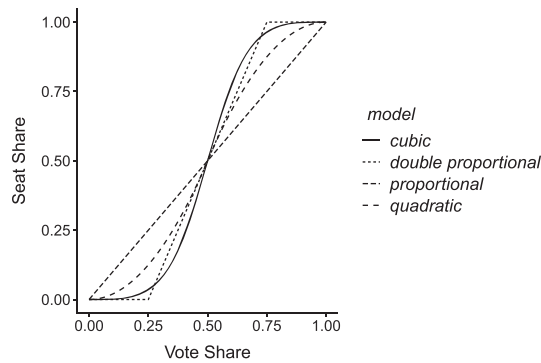
**Figure 1.** Commonly employed seats-to-votes curves.

through the point (0.5, 0.5).) Efficiency Gap analysis is equivalent to an analysis of how far the actual seat share-vote share combination lands from the line of double-proportionality without any consideration of counterfactual election results.[13]

The second category of metrics consists of measures based on the distribution of vote shares across seats. Two popular approaches are the *mean-median difference,* which flags maps in which two measures of central tendency in the distribution diverge (McDonald and Best 2015); and *declination*, an approach that looks for unexpected jumps in the vote share between districts won by Republican and Democratic candidates (Warrington 2018).

Justifications of partisan and asymmetry and bias and the Efficiency Gap generally turn on a notion of fairness grounded in the principle of anonymity, but as we detail below "fairness" in this case refers to the electoral prospects of political parties and not the fortunes of individual citizens. The mean-median difference and declination measures would appear to be geared more toward identifying maps that deviate in important respects from what we might expect in the absence of partisan considerations in map drawing. This brings up an important issue that is related to, albeit distinct from, our overall argument: a common approach taken by plaintiffs and amici in districting cases is to argue that a map is problematic because one or measures deviate from historical norms. This would seem to conflate the harm *from* gerrymandering on the one hand with culpability *for* gerrymandering on the other.

Establishing culpability is, in effect, a causal attribution problem: a high degree of partisan bias, e.g. could emerge because of deliberate strategic action on the part of partisan actors. But it could also emerge unintentionally owing to the spatial distribution of voters (Chen and Rodden 2013). A prominent example, which we return to in our empirical analysis, is the "Massachusetts problem" (Duchin et al. 2019). Roughly 1/3 of voters in that state tend to vote Republican. But because they are not geographically concentrated in any one part of the state, but rather uniformly distributed across it, it is virtually impossible to draw a map that makes any of its nine congressional districts reliably Republican. In light of the problem of unintentional gerrymandering, scholars have recently promoted *ensemble* methods as a more reliable way to establish culpability. Rather than compare a measure of gerrymandering to some absolute standard, these scholars instead advocate in favor of comparing the measure applied to a given map to the distribution of the measure calculated for hundreds of thousands of maps simulated under putatively neutral criteria. Rather than ask whether partisan bias or Efficiency Gap for a particular map is large in absolute terms, the relevant question is whether the map is an outlier relative to the distribution of counterfactual maps. In our empirical analysis below, we employ an ensemble method to assess maps using the measure we derive in the next section.

## 2.  CONCEPTUAL FOUNDATIONS FOR A THEORY OF REPRESENTATIONAL FAIRNESS

In the prior section, we described how canonical diagnostic measures of partisan gerrymandering conflate culpability for gerrymandering with harm from gerrymandering. In this section, we argue that

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[13]   Indeed, proponents of the Efficiency Gap consider the absence of reliance on counterfactuals to be a main selling point of the measure (McGhee 2014; Stephanopoulos and McGhee 2015).

in addition to this elision, much of the lack of clarity stems from a failure of both political and legal analysts to clearly delineate an account of harm to individual voters from certain district maps, the protection from which voters ought to be reasonably and equally entitled. This confusion emerges primarily from a failure to distinguish between two very different schools of thought concerning the nature of representation: what contemporary political scientists refer to as the distinction between *collective* representation and *dyadic* representation (Weissberg 1978; Ansolabehere and Jones 2011). Whereas collective representation conceives of representation primarily in terms of the relationship between the legislative assembly as a whole and the political community as a whole, dyadic representation conceives of it in terms of the agency relationship between individual representatives and the constituents who exercise direct electoral control over them.

In this section, we begin by summarizing these different accounts of representation before demonstrating how one can derive a class of measures for diagnosing partisan gerrymandering based on the seats-to-votes curve as deviations from a social optimum prescribed by a generalized model of collective representation. We also highlight some of the challenges of moving within the collective representation framework to an analysis of fairness grounded in equal protection. We then describe the Court's repeated rejection of the collective account.

From our discussion of collective representation, we proceed to describe the dyadic model, highlighting the embeddedness of the dyadic account in the political theory of both the proponents and opponents of the 1787 US Constitution. To fix intuition, we offer a qualitative description of the formal, game-theoretical model that underlies our analysis. (The details of the formal model appear in the Appendix.) The model gives rise naturally to a measure of dyadic RD, whose properties we explore in the last part of this section.

## 2.1 Collective Representation

### 2.1.1 Background

The model of representation that perceives a legislative body as standing for the political community as a whole has its origins in medieval political thought, appearing later in the writings of Edmund Burke and John Stuart Mill (1861). A collective theory of representation also features prominently, though as we discuss below not exclusively, in the political philosophy of the framers: e.g. in Federalist #10 Madison writes, "The effect of the first difference [between pure democracy and republicanism, which Madison earlier defines in terms of representative government] is … to refine and enlarge the public views, by passing them through the medium of a chosen body of citizens, whose wisdom may best discern the true interest of their country."

### 2.1.2 Collective representation and the seats-to-votes curve

Appeals to proportionality generally stem from democratic theory and a preference for favoring "equal" representation in order that the disparate voices within a political community be given voice, that the legislature as a corporate entity better represent the community. To the extent that our concern is with the benefits of representation and their unequal distribution, one might wish to ask whether a normative justification for proportionality (or other seat-to-vote curves) in a two-party system might be formally rooted in a more general theory. The answer is that it can be tied to the theory of collective representation, but with a critical limitation that relates directly to the difficulties this theory has encountered in court.

In particular, assume that a voter's interests are better served insofar as her preferred party has a greater share of seats in the legislative body. From the perspective of that voter, it makes no difference whether those seats include the one assigned to the voter's own district or not. Suppose, therefore, that the benefit to every member of a party is increasing in the seat share of that party but at a diminishing rate: in particular, suppose that for a member of party $A$ the benefit of seat share $s$ is equal to $s^\alpha$, where $\alpha$ is some number lying between zero and one. Likewise, the benefit for a member of party $B$ of seat share $1 - s$ is $(1 - s)^\alpha$. The *aggregate* welfare (weighted by the share of voters preferring each party) is $v s^\alpha + (1 - v)(1 - s)^\alpha$.

It is straightforward to demonstrate that as $\alpha$ approaches zero,[14] the seat share that maximizes aggregate voter welfare collapses to the vote share itself: i.e. $s = v$ or pure proportionality.[15] Moreover, this

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[14]   This corresponds to a situation in which the marginal benefit of one additional representative is initially quite large, but decreases rapidly.

[15]   Proportionality as an optimum may also be derived from a model in which the diminishing returns from collective representation follows a logarithmic form (i.e. log(s), with the benefit to someone from party $B$ equal log(1 − s)).

formulation can generate all manner of different seat-to-vote curves depending on the value of $\alpha$: e.g. if $\alpha = 2/3$, the optimal curve is Tufte's cubic function; if $\alpha \approx 0.45$, the optimal curve is nearly identical to Barton's quadratic.[16] As $\alpha$ approaches one, the curve stays close to $s = 0$ up until $v = 0.5$, and then jumps up close to $s = 1$.[17] While our general formulation does not fit the "double-proportionality" seat-to-vote curve implied favored by Stephanopoulos and McGhee in their Efficiency Gap analysis, it is not hard to imagine some alternative derivation that would rationalize *EG* on utilitarian grounds in a collective representation context.

Of course, much of the controversy surrounding partisan gerrymandering concerns unfairness, unequal protection, and disparate impact rather than welfare maximization. It is here that the theory of collective representation encounters challenges because the notion of disparate impact on voters in the collective representation context is incoherent. If the benefit to a party *A* voter from seat share $s$ is $s^\alpha$ and the benefit to a party *B* voter is $(1 - s)^\alpha$, then the only way to minimize the disparity between the welfare of the two voters is to set the seat share to $1/2$ *irrespective of the ratio of party A to party B voters in the state*. In other words, to the extent that it requires evidence of disparate treatment between classes of voters, an equal protection rationale for reliance on a seat-to-vote curve collapses to a *reductio ad absurdum*.

A partial exception to this is the Efficiency Gap, which has become the standard metric for evaluating partisan gerrymanders. But sustaining the exception comes at a serious cost: specifically, it requires (a) examining the disparate impact not to different classes of voters but to the parties themselves and (b) defining the objective of the parties not directly in terms of maximizing seats but indirectly in terms of minimizing *wasted votes*. Votes for a party may be wasted in one of two ways: either they are "cracked" (divided into districts into which they cannot form the majority necessary to elect a legislator) or "packed" (placed into districts in numbers considerably in excess above 50 per cent + 1 as a means of limiting their electoral impact.) By construction, the total number of wasted votes in a state is equal to 50 per cent of the total votes. Stephanopoulos and McGhee define the Efficiency Gap as the number of wasted Democratic votes less the number of wasted Republican votes, divided by the total number of votes.

The Efficiency Gap is a disparity metric, but it reflects the disparity between the electoral fortunes of *parties* rather than the electoral fortunes of *voters*. In a recent paper, Gordon and Yntiso (2024) demonstrate that seemingly counterintuitive policy prescriptions stemming from Efficiency Gap analysis result from a form of base rate neglect: in other words, the Efficiency Gap fails to account for the underlying distribution of Democrats and Republicans. Nagle (2017) develops a "voter-centric" variant of the Efficiency Gap where, in its simplest form, the voter-centric disparity is minimized along a proportional seat-to-vote curve. In exploring different properties of this measure, Gordon and Yntiso demonstrate that versions that differentially weight the harm from packing and cracking may permit departures from proportionality. However, a bigger issue remains: whereas it is clear that a wasted vote harms a party, it is not immediately clear whether there is harm to a specific voter associated with being wasted. Particularly in a framework that prioritizes collective representation, a voter may be content to be wasted herself if that waste is part and parcel of a broader scheme offering greater seat share for her party as a whole.

### 2.1.3 The Court's rejection of collective representation

Notwithstanding our ability to reverse engineer a welfarist account of assessments of partisan gerrymandering based on a coherent theory collective representation, the exercise still runs up against the Rucho majority's jaundiced perspective on collective representation accounts, which (1) holds that there is no historical basis for a belief that the Constitution "guarantees a certain degree of representation to political parties";[18] (2) rejects a conviction that the greater the departure from proportionality, the more suspect an apportionment plan becomes;[19] and (3) rejects the idea that "vote dilution" standards that

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16   As far as we are aware, we are the first to normatively ground a derivation of seat-to-vote curves this way.

17   One way of thinking about what the $\alpha$ parameter represents is a welfarist interpretation of the normatively appropriate level of *responsiveness* of the seat-to-vote curve.

18   "The Constitution supplies no objective measure for assessing whether a districting map treats a political party fairly," *Rucho v Common Cause* at 708. "The Framers would have been amazed at a constitutional theory that guarantees a certain degree of representation to political parties," (*Id.*).

19   *Davis v Bandemer*, 478 U.S. 109, 145–147 (1986) (J. O'Connor concurring) ("Nor do I believe that the proportional representation towards which the Court's expansion of equal protection doctrine will lead is consistent with our history, our traditions, or our political institutions" … "If members of the major political parties are protected by the Equal Protection

require that each vote carry equal weight might imply that each party must be influential in proportion to its supporters.[20]

At base, the Court's perspective reflects a discomfort with the idea of evaluating a system of representation that is inescapably rooted in geographic districts using diagnostics that treat districts and their boundaries as a mere inconvenience rather than as an intrinsic feature. Taking seriously the idea of representation in a system with districts and deriving measures of representational harm specific to such a context requires an alternative normative account.

## 2.2 Dyadic Representation
### 2.2.1 Background

In contrast to the collective account of representation described above, a *dyadic* model of representation conceives of the legislature as a constituent assembly of agents of separate constituencies. This model has its origins in medieval political theory (Lagarde 1937) and in the representative assemblies in pre- and early modern Europe (Blockams 2008; Stasavage 2020). Variants also appear in Hobbes' *Leviathan* (1651) and in the writings of Edmund Burke (1774) (who, recall, also wrote on the collective account as well).

A dyadic conception of representation may be found in the writings of both supporters and opponents of the constitution. In Federalist #57, Madison anticipates that representatives "will enter into the public service under circumstances which cannot fail to produce a temporary affection at least to their constituents," and that an "habitual recollection" of their dependence on the favor of their constituents will be generated via periodic elections. Antifederalist sentiment is most clearly aligned with the dyadic theory of representation, something seen most clearly in the oft-expressed concern that the ratio of constituents to representatives was far too low to effect adequate representation of disparate local interests.[21]

The twentieth-century theorist most closely associated with advances in the theory of dyadic representation is Hannah Pitkin, who defines representation as the "making present *in some sense* of something which is nevertheless *not* present literally or in fact" (Pitkin 1967). More recently, normative theorists have proposed different approaches to categorizing the criteria one might employ in evaluating dyadic representational relationships (e.g. Mansbridge 2009; Rehfeld 2009; Dovi 2006).

A research tradition that has evolved in parallel to that described above concerns accountability in dyadic electoral relationships. Much of the research in this tradition entails the articulation of game-theoretic models that seek to capture different but essential aspects of the principal-agent relationship between the voters and their representatives (*see* Ashworth 2012 for a review). Some critical features common to these models are (1) uncertainty, either about the qualifications or preferences of candidates or the (possibly costly) actions they might take when in office; (2) the motivation of the officeholder of remaining in office; and (3) election, at which points voters may cast their ballots for or against an incumbent after observing some (possibly noisy) indicator of performance in office.

It is in the context of this research tradition that Ansolabehere and Jones (2011) frame the nature of the dyadic relationship between representatives and constituents in stark terms: the relationship emerges because constituents wish to select the "right type" of politician and reward officeholders for the effort they exert. They refer to these phenomena, respectively, as the "selection" and "officeholder" effects. Selection pressures may emerge not merely on the basis of partisanship, but on other features of the legislator as well, including values, social characteristics, personalities, and policy congruence.

Clause from dilution of their voting strength, then members of every identifiable group that possesses distinctive interests and tends to vote on the basis of those interests should be able to bring similar claims.")

[20] *See Gill v Whitford*, 585 U.S. 48 (2018) ("This Court is not responsible for vindicating generalized partisan preferences"); *Bandemer v Davis*, 603 F.Supp 1479, 1492 (S.D. Ind. 1984) ("Political groups themselves do not have an independent constitutional claim to representation. That is, the Constitution does not guarantee proportional representation, but the Constitution does prohibit state action that inhibits an individual's right to vote.") (internal quotations omitted); *Wesberry v Sanders*, 376 U.S. 1, 8–9 (1964) ("To say that a vote is worth more in one district than in another would not only run counter to our fundamental ideas of democratic government, it would cast aside the principle of a House of Representatives elected 'by the People,' a principle tenaciously fought for and established at the Constitutional Convention. The history of the Constitution, particularly that part of it relating to the adoption of Art. I, §2, reveals that those who framed the Constitution meant that, no matter what the mechanics of an election, whether statewide or by districts, it was population which was to be the basis of the Hose of Representatives."); *Reynolds v Sims*, 377 U.S. 533, 562 (1964) ("It would appear extraordinary to suggest that a State could be constitutionally permitted to enact a law providing that certain of the State's voters could vote two, five, or ten times for their legislative representatives, while voters living elsewhere could vote only once. And it is inconceivable that a state law to the effect that, in counting votes for legislators, the votes of citizens in one part of the State would be multiplied by two, five, or ten, while the votes of persons in another area would be counted only at face value, could be constitutionally sustainable.")

[21] A prime example comes in Melancton Smith's speech to New York's ratifying convention. *See* Storing, op. cit., at 340.

When evaluating performance in office, constituents may take into account not just roll call voting, but the ability of legislators to address constituent problems and advocate for local interests. Hill, Jordan, and Hurley (2015), echoing Miller and Stokes (1963), refer to dyadic representation as fundamental to democratic government.

Critically, the dyadic model allows us to reframe the implicit *legal* question at the heart of litigation concerning partisan gerrymandering. Rather than ask "How are the Democrats (Republicans) adversely affected *collectively* by map $\mathcal{M}$ relative to Republicans (Democrats)?" we wish to ask, "How is the typical individual X adversely affected *individually* by map $\mathcal{M}$ relative to the typical individual Y?" where X and Y might be Democrats and Republicans, but might also allow for comparisons between, *inter alia*, urban vs. rural voters or Black vs. White voters. Viewed in this way, a logical flaw in Roberts' one-person-one-vote analogy is made transparent. Specifically, writing for the majority in *Rucho* Roberts draws a comparison between malapportionment and partisan gerrymandering cases in the following way: "More fundamentally, 'vote dilution' in the one-person, one-vote cases refers to the idea that each vote must carry equal weight. In other words, each representative must be accountable to (approximately) the same number of constituents. That requirement does not extend to political parties." The mistake is to interpret an "if" (i.e. if districts are malapportioned then votes don't carry equal weight) as an "if and only if" (i.e. that malapportionment implies unequal weight *and* unequal weight implies malapportionment). As we will make clear, malapportionment is just one way to dilute an individual vote, *even accepting that there is no constitutional requirement that parties (as collective entities) be treated equally*.

### 2.2.2  A heuristic model

*Preliminaries.* Here, we outline a formal model that captures benefits and harms in a dyadic representational context, expanding on the hypothetical example in this paper's Introduction. Technical details appear in the Appendix. Our model is sufficiently flexible to account for a variety of different electoral contexts, e.g. ones in which partisan polarization is high or low; where officeholder benefits (and thus the incentive of legislators to work hard on behalf of their constituents) are high or low; and in which incumbency advantage proves a vital component to be considered. Our model illustrates the subtle ways that the harms from partisan gerrymandering interact with each other and belie the over-simplified classification of districts as either "cracked" or "packed." At the same time, while our model is flexible, it is still intended to be heuristic: there are many alternative models of dyadic representation that one could envision and that could be applied in a similar way to the one described herein.

Specifically, our analysis focuses on two different harms to voters. First, voters are harmed when they live in a district where it is very unlikely that they will be represented by someone who shares their values—harm from "selection" effects, employing the parlance above. This harm is traditionally associated with voters who are "cracked"—i.e. divided into districts where they make up only minorities in each, although the probability this harm is realized may vary depending on the size of the minority in question. Second, voters are harmed when they are represented by somebody who shirks their responsibilities—harms from "officeholder" effects. This harm may befall both supporters and opponents of an incumbent and is likely to be minimized when the district is more competitive.

In the model, selection and officeholder effects may interact in subtle ways. For example, in a district with a supermajority of voters from one party, a large number of voters will get to enjoy representation by a like-minded legislator; however, that legislator may lack incentives to work hard on their behalf. Alternatively, a district with a sizeable partisan minority approaching 50 per cent may not have a likeminded representative at time $t$, but may have a shot at electing one at time $t+1$; moreover, a fear of that eventuality on the part of the incumbent will create the motivation to serve constituents—which may benefit the minority as well as the majority. On the other hand, if voters are ideologically polarized, then even a 49 per cent minority may never have a shot at electing a copartisan. These subtleties belie a simple categorization of districts and voters into "cracked" and "packed."

In order to explore these subtleties, we articulate a conceptual model that (1) captures the effects of these competing considerations on the welfare of voters in a way that (2) yields a measure of representational welfare disparity between different classes of voters (and is thus potentially amenable to being employed to assess equal protection claims). While the approach we adopt is sufficiently flexible as to admit a variety of different classes (Democratic vs. Republican, white vs. nonwhite, urban vs. rural, etc.), we will focus our attention on the current application to partisan differences.

Our model is closely related to a class of formal models in political science and economics called "career concerns" models (Holström 1999; Persson and Tabellini 2022; Ashworth 2005). For mathematical parsimony, we model the electorate within a district as a continuum of voters, each of whom may

be described by their partisanship and the intensity of their partisan attachment. The model allows for two interpretations of the latter. Per the first, partisan attachment is a kind of social identity (though see Green, Palmquist and Schickler 2002). Per the second, voters align with parties as a function of relative congruence with party (or candidate) platforms given platform divergence between the candidates (*see, e.g.* Wittman 1983; Calvert 1985; Alesina 1988).

Higher *average* levels of partisan attachment correspond to greater degrees of polarization between the parties, which may signify affective or policy polarization. Politicians know the normal vote in their districts (Converse 1966): i.e. what proportion of voters would vote for a Democrat or Republican if voters cast their ballots purely on the basis of partisan affiliation. They also know the level of polarization (conceived of as the degree of partisan attachment of a typical voter).

*Informal description.* There is an incumbent and a challenger, each of whom is characterized by their partisan affiliation and level of competence. While candidates and voters don't know their competence levels with certainty (the game is one of "symmetric uncertainty"), they do have beliefs about the distribution of those levels. They can also learn about an incumbent competence from observed performance in office. The incumbent's performance is a function of their competence, but also their effort and a random shock (i.e. luck or features of the political system otherwise outside of their control). To capture incumbency advantage in a simple and intuitive way, we allow voters' beliefs to be more *ex ante* favorable toward incumbents than challengers.

To capture the relevant dynamics, the model adopts the common simplifying approach of featuring two periods of play, and the sequence of play is as follows: first, nature determines the competence levels of the incumbent and challenger, the normal vote in the district, and the partisanship of the incumbent. Next, the incumbent chooses a level of effort, which, together with competence and luck, determines a level of performance. Third, voters observe performance and cast votes for the incumbent or a challenger from the opposing party. Fourth, the second-period incumbent (either the previous incumbent or the victorious challenger) chooses a level of effort. Finally, payoffs are realized.

When choosing their level of effort, incumbents are motivated by a desire to retain office ("intrinsic officeholder benefits," which may vary across individuals or electoral contexts) and avoid the cost of effort. Voters are motivated by their desire to elect a copartisan (scaled by their degree of partisan attachment) but also by a desire to elect a competent incumbent who will generate high levels of performance in the future. Importantly, voters in the model are not "fooled" by high effort into believing that a politician is more competent than she is; instead, they correctly conjecture the level of effort the politician will invest and discount performance accordingly when assessing the incumbent's competence. At the same time, voters benefit from that effort, insofar as it generates more favorable results.

*Insights and measurement.* The model yields the following:

(1) An expression for the equilibrium probability that a Democratic or Republican candidate wins as a function of the normal vote in the district, the degree of partisan polarization, and the extent of incumbency advantage.
(2) An expression for equilibrium level of incumbent effort as a function of the value of retaining office, the normal vote in the district, partisan polarization, and the extent of incumbency advantage. The more competitive the district, the stronger the incentive of the incumbent to work hard on behalf of her constituents.
(3) Expressions for net "representational benefit" to Democratic and Republican voters as functions of 1 and 2, from which we may derive measures of the aggregate welfare of a state's voter's corresponding to a given map and measures of the disparity between the average welfare of voters from different political parties.

The benefits the typical voter receives from selection and officeholder effects is captured by our concept of *representational benefit*. For example, consider a Democratic voter in a very heavily Republican district. That voter is in some sense harmed twice: he has no hope of electing a copartisan who shares his values; moreover, because the district isn't competitive the incumbent will have little reason to work hard on his behalf. By contrast, consider a Republican in the same district. That voter will rest easier knowing that her representative will share her values more than one from the opposite party. However, she might still suffer from the incumbent's disinclination from working hard because she resides in a safe district. How these effects interact depends heavily on two key parameters: the level of polarization and the benefit of holding office. Fig. 2 shows how an individual voter's representational
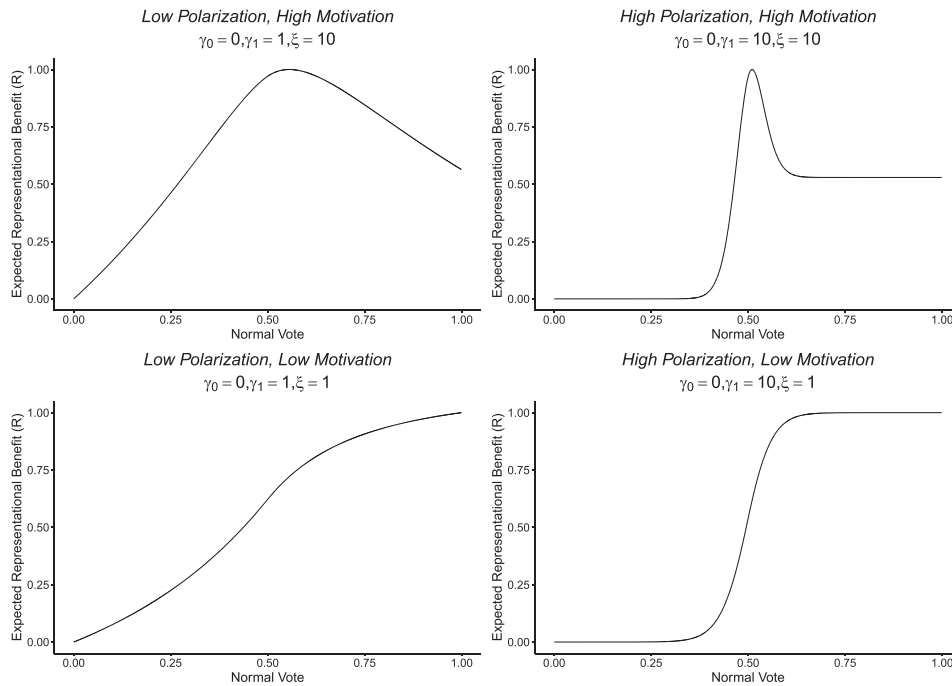
**Figure 2.** Representational welfare is given different parameter values in the model.

welfare may vary as a function of the normal vote in favor of his preferred candidate, depending on those two parameters.[22]

Consider the cell in the upper left, which corresponds to an environment of low polarization and high office benefits. In this circumstance, the competitiveness of the district (which is maximized when the normal vote is 50 per cent) benefits Democratic and Republican voters alike. A voter would prefer to be in the majority (note that the curve is maximized above 50 per cent) but is harmed if the majority is too large—*not* because of the collective harm of packing, but because of the dyadic harm of incumbent disinvestment from working on behalf of her constituents. By contrast, consider the cell in the lower right. In that circumstance, polarization is high and officeholding benefits are low. Because of the latter, incumbent effort on behalf of constituents will tend to be minimal, and what voters will care about most of all is electing a copartisan. This is reflected in the picture—a higher normal vote on behalf of one's copartisan leads to a higher payoff for the voter—there is no offsetting premium to be had from a more competitive district.

*Comment.* Our model is intended to be heuristic and does not capture every aspect of the dyadic relationship between representatives and constituents. One obvious feature that does not appear in the model is *partisan primaries*. In evaluating the consequences of that omission, consider that partisan primaries tend to produce ideological divergence between candidates from different parties. In expectation, this will benefit the typical copartisan of the incumbent, but other features of the model, e.g. partisan polarization, can already capture this effect.

A second feature of the model to which one might object is its stark focus on dyadic benefits to the exclusion of collective ones. Of course, voters benefit both from their affinity toward and the performance of their own incumbent as well as from the composition of their state's delegation or the legislature as a whole. Adding a term for collective benefits in the voter's utility function would be straightforward; to make the intuition for dyadic representation as clear as possible, we do not do so here.[23]

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[22]    For ease of interpretation, we shut down the incumbency advantage channel both in this Figure and in the empirical analysis that follows.

[23]    In practice, the inclusion of a term for collective benefits would penalize maps that diverge from some seats-to-vote curve.
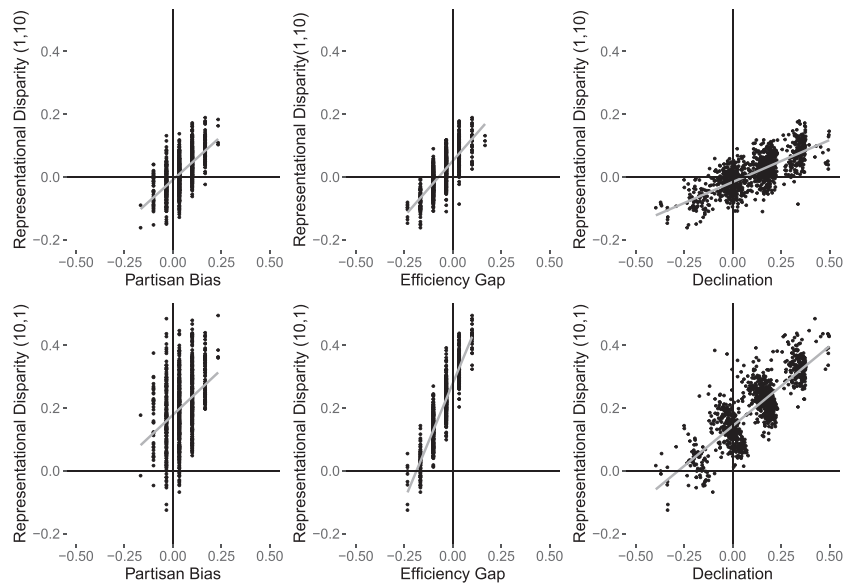
**Figure 3.** Comparing representational disparity in simulated data with three commonly used partisan gerrymandering diagnostics. For the top panels, polarization = 1 and officeholder benefits = 10. For the bottom panels, polarization = 10 and officeholder benefits = 1.

### 2.2.3 *Representational disparity*

Our notion of representational benefit gives rise to a disparate impact measure: Dyadic *RD*. *RD* is simply the difference in the average representational benefit for one vs. another type of voter, e.g. Democrat vs. Republican, normalized so that the underlying representational benefits are bounded between zero and one. To show how *RD* compares with traditional diagnostic measures of partisan gerrymandering, we created a completely artificial state with a delegation of fifteen seats. Sixty per cent of the state's voters are Republicans and 40 per cent are Democrats. We then randomly generated 1,000 "maps." *Just for the purpose of this exercise*, we impose no geographic constraints—any voters can be grouped with any others as long as the districts are equally populous and the statewide vote share is 60–40. All such groupings are equally likely.[24] Fig. 3 displays scatterplots comparing *RD* at two parameter values (low polarization/high motivation in the top three panels; high polarization/low motivation in the bottom three) with three commonly deployed diagnostic measures: partisan bias, efficiency gap, and declination. Lines of best fit appear in blue on each panel.

The first thing to note about the figure is that across the measures, there is a strong, positive correlation between *RD* and the "classic" alternative. Insofar as partisan bias and Efficiency Gap may be derived from a model of collective representation, this suggests that collective and dyadic representation are not completely disjoint concepts.[25]

Second, in the graphs comparing *RD* with partisan bias and Efficiency Gap, the data take the form of seven bands corresponding to discrete values of the canonical measures. This reflects the underlying connection between these measures and the seat-to-vote curve. Each band corresponds to a different seat share for Republicans: in our 1,000 simulated maps, Republicans had majorities in anywhere from seven (47 per cent of fifteen) to thirteen (87 per cent of fifteen) districts.[26] Partisan bias and efficiency gap each take on a distinct value corresponding to each of those seat shares. By contrast, *RD* varies continuously—and sometimes dramatically—between maps, *even holding seat share constant*.

Finally, the lower tier of panels in the figure demonstrates that for sufficiently high polarization, we should not expect RD to equal zero in maps that one would, upon inspection, consider plausibly "fair."

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[24] In technical terms, the procedure generates *K*-dimensional seat share vectors using a rejection sampling algorithm written by the authors.

[25] Declination, by contrast, is a measure of irregularities in the distribution of vote shares across districts that cannot be derived from either a collective or dyadic representational theory.

[26] The median was ten. Seat shares of seven and thirteen are extraordinarily rare.

*Ceteris paribus*, in a state that is 60-40 Republican voters, you'd rather be a Republican no matter the map. So, the relevant question shouldn't be, "how close is representational disparity to zero." Rather, it should be "how does representational disparity from this map compare to representational disparity from other maps?"

## 3. EMPIRICAL ANALYSIS OF DYADIC REPRESENTATIONAL HARM

In the preceding section, we considered the range of dyadic representational disparities that are possible without regard to political geography or redistricting criteria. In the following section, we present an empirical analysis of the representational outcomes that are possible given actual voting patterns in recent US Congressional elections. Section III.A. describes the two principal ingredients of that analysis: (1) an unbiased measure of the underlying distribution of partisan loyalties and (2) a well-described distribution of feasible counterfactual maps. Section III.B. demonstrates how collective and dyadic representational harms may differ using four illustrative case studies: Massachusetts, Florida, Pennsylvania, and New York.

### 3.1 Data

#### 3.1.1 *Precinct-level election data*

Local measurements of partisan preferences are vital for calculating district vote shares and representational harm across redistricting plans. To that end, we collect precinct-level averages of the two-party vote share across the Presidential, Senate, and Gubernatorial races held between 2016 and 2020. This measure of the "normal vote" aims to isolate voter preferences from idiosyncratic factors, such as election cycle, election office, individual candidate, or enacted map, that may influence an observed vote outcome.

#### 3.1.2 *Ensemble-based analysis*

To establish a reasonable baseline of fairness given a state's political geography, overall partisanship, and redistricting rules, we turn to the second principal ingredient of our approach: a well-calibrated distribution of feasible counterfactual maps. We generate an ensemble of valid plans to simulate the range of possible disparities attributable to the spatial distribution and partisanship of voters. A proposed plan that deviates too far from that range furnishes evidence in support of a claim that inequality of representational benefits stemming from a partisan gerrymander is particularly egregious.

Despite its increasing prominence, ensemble-based redistricting analysis poses numerous methodological challenges. First, an unrepresentative sample of redistricting plans may bias inferences about representational harm, just as a survey of the readers of a right-wing tabloid may bias inferences about the attitudes of the general public. Although a complete enumeration of all potential maps is infeasible, Markov Chain Monte Carlo (MCMC) algorithms have been shown to generate representative samples from a well-defined target population of redistricting plans (Fifield et al. 2020; Deford, Duchin, and Solomon 2021; McCartan and Imai 2023). The target population is defined by weighting some hypothetical plans—in particular, plans that satisfy each state's legal redistricting criteria—over others (Duchin and Spencer 2021). A Markov Chain is a sequence of maps designed to randomly "walk" over that population. Feasible plans are encouraged, and infeasible or unconstitutional plans are discarded in each step of the process, so that the MCMC ensemble will ultimately approximate the target population of potential maps, irrespective of the initial starting map.

We use the Recombination algorithm, *ReCom*, developed by Deford, Duchin, and Solomon (2021) to generate an ensemble of 500,000 valid redistricting maps for Massachusetts.[27] *ReCom* presents several advantages over alternatives: first, the algorithm is highly efficient, reassigning hundreds of precincts at each step. Further, all steps in a *ReCom* chain pass validity checks, obviating the need to discard initial plans. Finally, *ReCom* achieves compactness by design, foreclosing any need to explicitly configure compactness targets.[28]

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[27] For the sake of illustration, our empirical analysis relies on the 499,533 simulated plans that produce one-party delegations. The 467 excluded plans include one Republican district; the highest vote margin detected in those Republican districts was 0.0145. We require districts to be contiguous, compact, have equal populations, and maintain precinct integrity. State law does not mandate contiguity or precinct preservation for congressional districts.

[28] As a robustness check (available from the authors upon request), we replicate our analysis using redistricting ensembles generated by the Algorithm-Assisted Redistricting Methodology (ALARM) Project (McCartan et al. 2022), available at https://alarm-redist.org/.
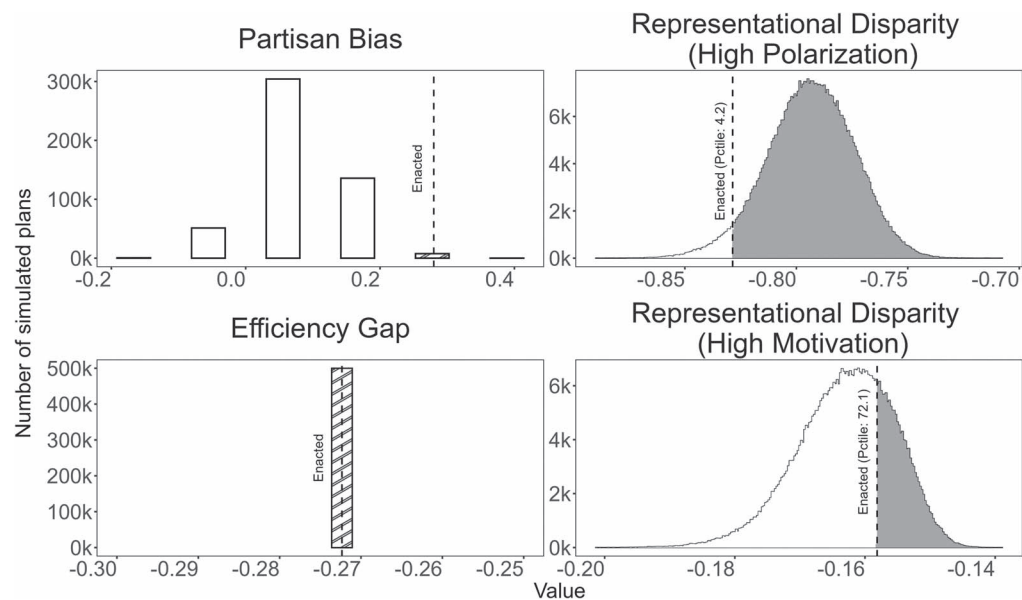
**Figure 4.** Collective and dyadic representational disparity metrics comparing Massachusetts' enacted plan with a sample of 500,000 simulated counterfactuals. *Note:* Shaded (unshaded) regions represent simulations that are more Republican (Democratic) than the enacted plan. Stripes denote simulated plans with values equal to the enacted plan.

## 3.2 Illustrative Examples

### 3.2.1 Massachusetts

We begin by evaluating Massachusetts' most recent congressional districting plan. As Duchin et al. (2019) point out, Republicans constitute 30 per cent to 40 per cent of the two-party vote in that state, yet they rarely form a numerical majority in the municipalities and precincts where they reside. This makes it nearly impossible to establish even a single Republican district. As we will demonstrate, significant variation in dyadic RD will emerge within the set of feasible districting plans in that state, even though common measures of collective harm, such as the Efficiency Gap, will judge those plans to be substantively indistinguishable. For example, each feasible plan equally disadvantages Republicans compared to a proportionality benchmark (1/3 Republican seats) or the "cube law" (1/9 Republican seats). This situation therefore offers a unique opportunity to show what distinguishes the approach from collective accounts. In particular, we can examine whether feasible congressional redistricting maps can achieve fairer dyadic representation than the implemented plan, given the spatial arrangement of voters in Massachusetts.

The histograms presented in the left panels of Fig. 4 plot partisan bias (top) and the Efficiency Gap (bottom) across simulated redistricting plans. Throughout our analysis, positive values of bias indicate that a map is more favorable to Republicans, while negative values imply that it is more favorable to Democrats. Predictably, the Efficiency Gap exhibits uniformity throughout the entirety of the ensemble, given that, in every feasible map, all Republican votes and Democratic votes exceeding the 50 per cent threshold are wasted. Partisan bias takes only six values across the ensemble, with the implemented plan (denoted by a dashed vertical line) ranking as one of the most pro-Republican plans in the ensemble. This suggests that in a world where Republicans will ordinarily not capture even a single seat, the enacted plan is nonetheless relatively favorable to them according to this metric.

The right panels of Fig. 4 demonstrate dyadic representational disparities arising from two distinct yet illustrative models of representation. The histograms capture values of these measures across our sample of *ReCom* simulations. Unsurprisingly, representational disparities consistently fall below zero in Fig. 4, reflecting the fact that one would generally prefer to be in the majority than the minority. Fig. 4 also highlights the value of our ensemble-based approach of comparing enacted plans with feasible maps rather than an absolute standard. The top right panel depicts disparity measures from a model of heightened partisan polarization and minimal incumbent motivation. Heightened

polarization increases representational advantage to constituents represented by legislators sharing the same partisan affiliation, while adversely impacting those voters represented by legislators of the opposing party. At the same time, low incumbent motivation reduces the benefit to Republican voters of being in more competitive districts. Compared to collective representation metrics that tend to feature a single or discrete range of representational outcomes, our dyadic approach reveals a continuum of potential representational disparities. Diverging from collective approaches, the high polarization parameterization of the dyadic representation model indicates that the implemented plan is in the fourth percentile, meaning it is more favorable to Democrats than 96 per cent of the entire ensemble.

We can contrast these findings with those derived from an alternative account that affords greater importance to officeholding benefits and less significance to polarization. As officeholding benefits increase, incumbents will be motivated to intensify their efforts in competitive districts. The bottom-right panel of Fig. 4 illustrates representational disparities derived under assumptions of high motivation and relatively low polarization. Democratic voters still consistently enjoy a favorable position across the spectrum of simulated redistricting plans, but Republicans now receive some representational benefit from incumbent effort. Our analysis reveals that the enacted plan is at the 72nd percentile, meaning that it is more beneficial to *Republican voters* than 72 per cent of the simulated plans.[29]

In sum, although Republican voters in Massachusetts are persistently unable to win a single seat and are, consequently, worse off in absolute terms under plausible representational assumptions, whether we judge the enacted map to be relatively favorable to Republican or Democratic voters compared to alternatives depends critically upon the assumptions we make about the benefits of representation.

### 3.2.2 *Florida and Pennsylvania*

Next, we apply our methodology to two states that differ from Massachusetts in important ways: Florida and Pennsylvania. Fig. 5 displays collective and dyadic measures for each state. First, note that in contrast to Massachusetts, the spatial arrangement of voters in these states permits a wide variety of seat shares for different maps. This is reflected in the multiple feasible values for both partisan bias and Efficiency Gap, as depicted in the left panels of the figure. Second, relative to our ensemble of simulated maps, implemented maps excessively favor majority-party voters—Republicans in Florida and Democrats in Pennsylvania.

Third, unlike in the case of Massachusetts, whether we conclude the enacted map is more pro-Republican or pro-Democratic on dyadic representation grounds (relative to the set of feasible alternatives) is not sensitive to our assumptions about polarization or officeholder motivation (right panels of Fig. 5).

No matter what those assumptions, we would reach a similar substantive conclusion for both states: the enacted plans are highly favorable to the majority. The issue is particularly acute in Florida, where the enacted plan generates a more pro-Republican disparity than 99.9 per cent of alternatives, despite Florida's constitutional injunction against partisan gerrymandering.[30]

### 3.2.3 *New York*

For our final illustrative example, we focus on New York. In the most recent cycle, an independent redistricting commission (IRC) first proposed two maps, which the heavily Democratic Legislature rejected before proposing its own map. The New York Court of Appeals struck down the legislature's plan and imposed its own, drawn by a special master. The court's plan, which was anticipated to shrink the number of Democrats elected to Congress, led to harsh criticism from prominent liberals in the state.[31] This set of circumstances gives us an opportunity to assess the relative merits of several different plans within the same state.

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[29] This finding can be explained, in part, by the fact that the enacted plan includes an unusually large number of competitive districts. Larger incumbent incentives favor Republican voters more in redistricting plans that concentrate Republicans into a few, competitive districts than in maps that spread Republicans evenly across several districts.

[30] As described above, Florida Const. Art. III §20 prohibits the drawing of districts "with the intent to favor or disfavor a political party or incumbent."

[31] *See, e.g.* House Democratic Leader Hakeem Jeffries, who published a statement criticizing the maps for undercutting the will of the people because they "were drawn by an unelected, out-of-town Special Master appointed by an inexperienced, right-wing Steuben County judge who was handpicked by extreme, partisan activists." Press Release, *Leader Jeffries Statement on the Adoption of New York State Congressional Map*, February 28, 2024. New York State Senate Deputy Leader Michael Gianaris also called the maps a "partisan attack by the Court of Appeals." Lindsay Shachnow, "State Government Is More Important Than It's Ever Been: Senator Gianaris on Upheaval at New York's Top Court, Redistricting, Bail Reform, & More," *Gotham Gazette* (July 26, 2022).
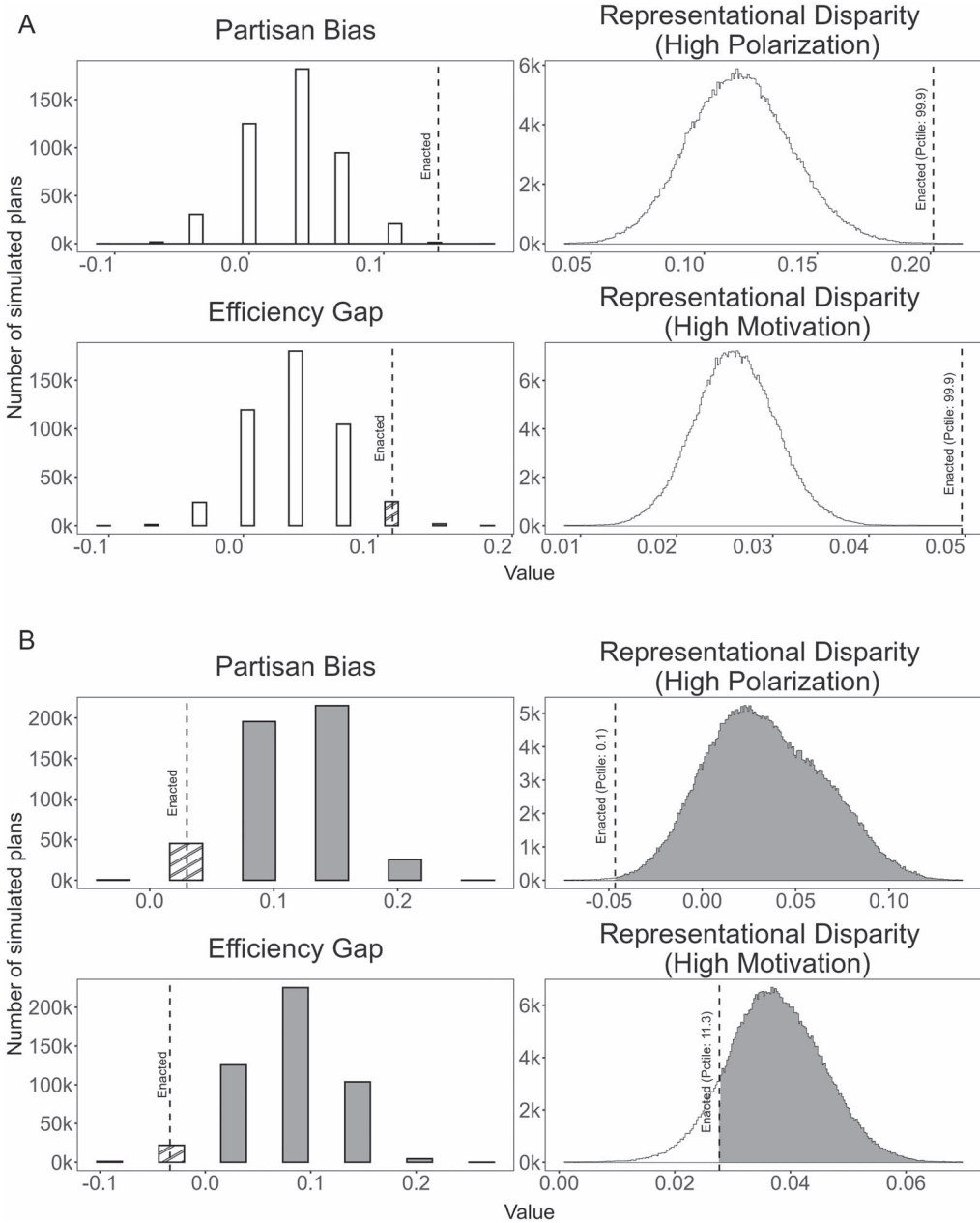
**Figure 5.** Collective and dyadic representational disparity metrics comparing Florida's and Pennsylvania's enacted plans to samples of 500,000 simulated counterfactuals. *Note:* Shaded (unshaded) regions represent simulations that are more Republican (Democratic) than the enacted plan. Stripes denote simulated plans with values equal to the enacted plan.
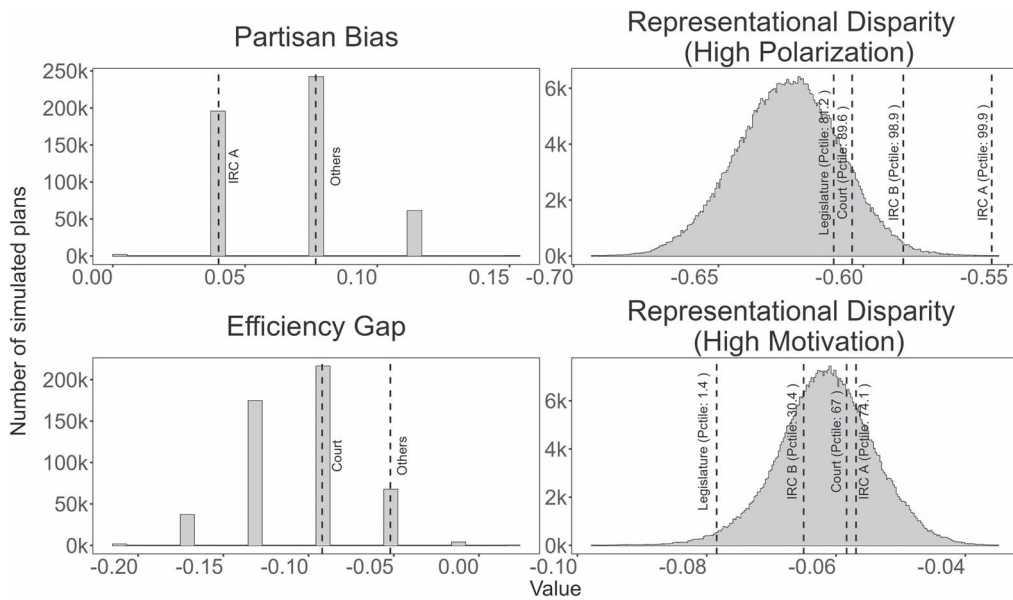
**Figure 6.** Collective and dyadic representational disparity metrics comparing four New York plans to a sample of 500,000 simulated counterfactuals. *Note*: For visual clarity, shading does not vary.

Fig. 6 replicates our analysis with reference to all four plans: the two plans from the Commission (labeled *IRC A* and *IRC B*), the Legislature's plan (*Legislature*), and the Court-drawn plan (*Court*).

Focusing on the top right panel (high polarization), we see that Democrats are highly favored across all maps in our ensemble, as indicated by the fact that the support of the distribution lies substantially below zero. Second, notwithstanding complaints from Democrats, on dyadic representational grounds, there is very little difference between the legislatively-enacted plan and the court-enacted plan. Moreover, both fall squarely within the range of feasible alternatives. By contrast, IRC A is relatively more favorable to Republicans.

Turning to the bottom right panel (high motivation), some interesting differences emerge. First, the support of the distribution is much closer to zero. This simply reflects the fact that New York Republicans, although still in the statewide minority, are much more likely to reside in, and benefit from, competitive districts. Second, we now see a substantial difference between the legislative and court plans. Specifically, whereas the legislative plan is far more favorable to Democratic voters than the lion's share of feasible alternatives, the court plan (along with the others) falls comfortably within the range of expected values given our assumptions.

### 3.3  General Takeaways

The foregoing analysis suggests that disparity metrics can sometimes (but critically not always) be highly sensitive to the assumptions one makes about a state's underlying political environment. In some cases (New York), changing those assumptions can dramatically affect our conclusions about whether a state is a particularly egregious outlier in terms of RD. In other cases (particularly Florida as our analysis suggests), our conclusions about the outlier status of a particular districting plan are highly robust to changes in those same assumptions.

Unlike the case with collective metrics, it would thus be imprudent to characterize a redistricting plan as an extreme gerrymander merely based on the expected/predicted partisan breakdown of its districts. The same map with the same partisan breakdown could be an extreme outlier among all possible maps or not, depending on underlying levels of polarization and assumptions about officeholder motivation.

In cases where our initial conclusions about the extent of gerrymandering harm are sensitive to these assumptions, identifying extreme gerrymanders necessarily becomes an exercise in hyper-localized analysis of voter behavior and the preferences of candidates and officeholders. These preferences may be captured by historical election data, state law, locally adopted redistricting principles, and even

survey data. This information can be crucial for performing a narrowly tailored assessment of the dyadic representational harms inherent in a given redistricting plan.

## 4. CONCLUSION

Partisan gerrymandering can threaten the health of a democratic republic, undermine good governance, and deprive individuals of free and fair representation. The harms of partisan gerrymandering cannot be fully understood without an appreciation of the way that a districting plan distorts the relationship between a voter and her representative. In a political system defined by geographic districts, this nuanced understanding requires a shift in focus from collective theories of representation to dyadic accounts.

Our analysis illustrates how quantifying representational harm in *dyadic* terms can open the door to Equal Protection arguments that center the individual voter—something that has been missing in federal and state litigation alike. Our measure also demonstrates how the interplay between features of the electoral landscape can dramatically affect our judgments of the nefariousness of a redistricting plan. In some cases, we observe representational disparities that favor one party over another to a degree not justified by the underlying political geography or redistricting criteria. In other cases, what might *look* like an unfair plan simply reflects contestable assumptions about factors such as polarization and incumbent motivation and less about the particular shape of any district.

The quest for judicially manageable standards in partisan gerrymandering cases is both necessary and urgent. The Supreme Court's refusal to adjudicate partisan gerrymandering claims has left a hole that state courts and independent commissions must strive to fill. The goal of this paper extends beyond convincing judges to consider challenges to partisan gerrymandering: our objective is to articulate a coherent account of representational fairness generally. By offering a microfounded perspective on gerrymandering that centers the individual voter, we hope to push legal scholars, policymakers, and the judiciary to reconsider how they approach issues of equal protection in the domain of democratic representation. As the nation grapples with the challenges of ensuring fair and equal representation for all its citizens, our work underscores the importance of developing robust, nuanced metrics and legal standards capable of safeguarding the democratic ideal against the distortions of partisan gerrymandering.

## Acknowledgements

## A. Appendix: Formal Model
### A.1 Primitives

In this section, we describe a simple formal model intended to capture three critical aspects of what it means to be represented well: to have a representative who is qualified or competent, who shares your values, and who works hard on your behalf. The ultimate aim is to derive a metric summarizing disparities in representation between different types of voters.

We begin by considering a single district. There are two parties $p \in \{D, R\}$. A district has a continuum of voters of measure 1. A fraction $\pi \in [0, 1]$ are affiliated with party $R$, and $1-\pi$ with party $D$. The exogenous parameter $\pi$, which is commonly known to all players, may be thought of as a representation of the *normal vote* in the district: i.e. the proportion of voters that would vote for a Democrat or Republican if voters cast their ballots purely on the basis of partisan affiliation (Converse 1966).

We leave voter party affiliation loosely defined, noting that it refers to a general ceteris paribus affinity toward the candidates of that party, rather than formal party registration. To capture this, we assume that each voter is characterized by an attachment parameter $a$ that ranges from $-A$ to $A$. Negative values of $a$, which are uniformly distributed on the interval $(-A, 0)$, denote attachment to party $D$, with larger negative values corresponding to greater partisan attachment and values closer

to zero implying weak attachment. Positive values of $a$, which are uniformly distributed on the interval $(0, A)$, correspondingly denote attachment to party R, with larger positive values corresponding to greater partisan attachment. Hence, parameter $A$ may be thought of as a shorthand representation of the extent of expected partisan polarization between randomly matched pairs of voters from the two different parties. $A$ is also commonly known to all.

There are two candidates: an incumbent $i$ and challenger $c$. Politicians are described by their partisan affiliation ($D$ or $R$) and by a type, $\theta_i$ or $\theta_c$. $\theta$ may be thought of as a summary of their underlying qualifications or competence in securing benefits (policy or local distributive goods) for their constituents. For example, the incumbent may have a knack for engaging in constituency service, helping to secure federal grants or contracts, or learning about the specific needs of constituents that they may apply effectively in Congress. We consider an environment of symmetric uncertainty in which neither the voters nor the politicians themselves know the precise values of $\theta_i$ and $\theta_c$, but all share the same prior distribution for those values.[32]

Specifically, we assume that $\theta_i$ is distributed normally with mean $\mu_i$ and precision (i.e. reciprocal variance) $\tau_i$. $\theta_c$ is distributed normally with mean zero and precision $\tau_c$. We allow $\theta_i$ to differ from zero to capture *selection-based incumbency advantage*: over a sequence of elections, voters may be expected to "filter out" lower-quality incumbents, and so one would reasonably expect $\theta_i$ to exceed $\theta_c$ on average (Ashworth and de Mesquita 2008). This of course is just one source of incumbency advantage; another might be the incumbent's access to the perquisites of office (Gordon and Landa 2009). In the context of this model, the latter source of the incumbency advantage will have an effect on representation similar to those of other components of the model, so we opt not to model it here to avoid adding superfluous clutter to the math.

The incumbent officeholder can exert effort, $e \in \mathbb{R}^+$, that, as with talent, also improves the welfare of her constituents. The outcome in period $t$ valued by all voters is

$$y_t = e_t + \theta_{i,t} + \varepsilon_t,$$

where $\varepsilon_t$ is a random shock drawn from a normal distribution with mean zero and precision $\tau_\varepsilon$.

The game unfolds as follows:

1. Nature draws $\theta_{i,1}$, $\theta_c$, $\varepsilon_1$, and the partisanship of the incumbent
2. Period 1 incumbent chooses effort $e_1$
3. Voters observe $y_1$, votes for incumbent or a challenger from the opposing party
4. Nature draws $\varepsilon_2$
5. Period 2 incumbent chooses effort $e_2$
6. Payoffs realized

Let $p_{i,t}$ denote the party of the incumbent in period $t$. The utility function for a voter from party $p$ with partisan attachment $a$ is given by

$$u_v(p, a) = \sum_{t=0}^{2} \left( y_t + \mathbb{I}\left( p = p_{i,t} \,|a| \right) \right)$$

where $\mathbb{I}(\cdot)$ is an indicator function. In words, the voter values the overall performance of the incumbent as well as being represented by a legislator from the same party, with the latter scaled by the degree of the voter's partisan attachment.

A critical value of $a$ is the district's median, i.e. the value of $a$ such that 50 per cent of voters have lower $a$ and 50 per cent greater values of $a$. We label this value $\tilde{a}(\pi, A)$.

**Lemma 1 (Identity of the median voter)** *The median voter has a partisan attachment of*

$$\tilde{a}(\pi, A) = \begin{cases} \frac{(2\pi - 1)A}{2\pi}, & \text{if } \pi \geq \frac{1}{2} \\ \frac{(2\pi - 1)A}{2 - 2\pi}, & \text{otherwise} \end{cases} \tag{A.1}$$

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[32] Symmetric uncertainty avoids complications that might arise if incumbents had incentives to signal their privately known type, a potentially interesting phenomenon in its own right but one that is not germane to the current inquiry.

**Proof**. Suppose $\pi \geq \frac{1}{2}$. Then the median voter is attached party $R$, and $\tilde{a}(\pi, A)$ is the solution to $\pi \int_{\tilde{a}}^{A} \frac{1}{A} dt = \frac{1}{2}$, given in the first line of Equation (A.1). Suppose $\pi < \frac{1}{2}$. Then the median voter is attached to party $D$, and $\tilde{a}(\pi, A)$ is the solution to $(1 - \pi) \int_{-A}^{\tilde{a}} \frac{1}{A} dt = \frac{1}{2}$, given in the second line of Equation (A.1).

The value of $\tilde{a}(\pi, A)$ when $\pi$ falls below 1/2 is negative: intuitively, when the normal vote skews toward party $D$ the median voter comes from that party. When $\pi$ exceeds ½ the median voter comes from party $R$. When $\pi = 1/2$ precisely, the two expressions are equal to each other, as are their derivatives, implying that the value of $\tilde{a}(\pi, A)$ is smooth and continuous over the full range of $\pi$.

The incumbent in the first period values being returned to office less the cost of the action $e$. Let $B \in \mathbb{R}^+$ denote the return to the incumbent to holding office, and $b = 1$ if the incumbent is reelected and 0 otherwise. The incumbent's utility function is given by

$$u_i(e_1, e_2) = B - c(e_1) + b(B - c(e_2)),$$

where $c(\cdot)$ is a strictly increasing convex function with $c(0) = 0$. The challenger's utility is given by

$$u_c(e_2) = (1 - b)(B - c(e_2)).$$

The solution concept is weak perfect Bayesian equilibrium. Additionally, we restrict attention to equilibria with weakly undominated strategies, which will permit us to focus on the unique equilibrium in which voters cast sincere votes for their favored candidates.

## A.2 Equilibrium

To solve for the equilibrium, first note that in the second period, neither the challenger nor the incumbent has an incentive to exert effort, which is costly but affords no electoral gains. Hence, $e_2^* = 0$. Thus, the value of the second-period outcome will be a function of the period 2 incumbent's type plus (mean zero) noise.[33]

Let $\hat{\theta}_{i,1} \equiv E[\theta_{i,1}|y_1]$, the voter's posterior expectation of the period 1 incumbent's type conditional on observing the outcome $y_1$. In equilibrium, voters will correctly conjecture the level of incumbent effort $e_1^*$, and discount their posterior beliefs on $\theta_i$ conditional on outcome $y_1$ accordingly. Using standard results on Bayesian updating, $\hat{\theta}_i$ is given by

$$\hat{\theta}_{i,t} = \lambda(y_1 - e_1^*) + (1 - \lambda)\mu_i \qquad (A.2)$$

where $\lambda \equiv \frac{\tau_\varepsilon}{\tau_\varepsilon + \tau_i}$. This has an appealing interpretation as the weighted average of the signal (discounted by the known effort level $e_1^*$) and the prior mean $\mu_i$.

The prospective expected utility to a voter from the party $p$ given partisan attachment $a$ of the incumbent being retained is given by

$$E[u_v(\text{retain}; p, a)] = \mathbb{I}(p = p_{i,1})|a| + \hat{\theta}_{i,1}.$$

Recalling that the expected competence level of the challenger, $E[\theta_c] = 0$, the expected utility to the incumbent being replaced is

$$E[u_v(\text{replace}; p, a)] = \mathbb{I}(p \neq p_{i,1})|a|.$$

Rearranging terms, a voter will vote to retain an incumbent from party $R$ if and only if $\hat{\theta}_{i,1} > -a$; and an incumbent from party $D$ if and only if $\hat{\theta}_{i,1} > a$. Note that by monotonicity, if this inequality is satisfied for $a'$ it is satisfied for any $a'' > a'$. Hence, the support of the median voter is necessary and sufficient

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[33] Readers unfamiliar with two-period models may object that it is unreasonable to assume the game ends after a second period—after all, shouldn't the second period incumbent want to exert effort to increase the likelihood of election to a third period? As with many two-period models, the second period is a convenient abstraction, the function of which is to induce sensible preferences and behavior in the first period (which is then the subject of our central inquiry). Specifically, the existence of the second period induces a situation in which ceteris paribus, voters prefer more highly-qualified incumbents when casting their ballots at the end of the first period, and incumbents are incentivized to work hard in the first period to enhance their electoral prospects.

for the incumbent to be retained, and the probability that this occurs is equal to $\Pr\left(\hat{\theta}_{i,1} > -\tilde{a}\left(\pi, A\right)\right)$ for an incumbent from party $R$ and $\Pr\left(\hat{\theta}_{i,1} > \tilde{a}\left(\pi, A\right)\right)$ for an incumbent from party $D$.

From Equation (A.2), $\hat{\theta}_{i,1} = \lambda\left(e_1 + \theta_{i,1} + \varepsilon_1 - e_1^*\right) + (1 - \lambda)\mu_i$. The *expected* voter posterior mean on the incumbent's type is $\lambda\left(e_1 - e_1^*\right) + (1 - \lambda)\mu_i$. The corresponding expected posterior precision is given by $(\tau_\varepsilon + \tau_i)$. Then the probability the incumbent is retained is given by

$$\Phi\left(\left(\tilde{a}\left(\pi, A\right)P_R + \lambda\left(e_1 - e_1^*\right) + (1 - \lambda)\mu_i\right)\left(\tau_\varepsilon + \tau_1\right)^{\frac{1}{2}}\right)$$

where $\Phi\left(\cdot\right)$ is the cumulative standard normal distribution function and $P_R$ takes the value of 1 if the incumbent is from party $R$; $-1$ if from party $D$; and zero otherwise.

The period 1 incumbent's optimization problem may be expressed as

$$\arg\max_{e_1}\left[\Phi\left(\left(\tilde{a}\left(\pi, A\right)P_R + \lambda\left(e_1 - e_1^*\right) + (1 - \lambda)\mu_i\right)\left(\tau_\varepsilon + \tau_1\right)^{\frac{1}{2}}\right)B - c\left(e_1\right)\right].$$

Taking derivatives and noting that in equilibrium, $e_1 = e_1^*$, we have the following first-order condition:

$$\frac{\tau_\varepsilon B}{\left(\tau_\varepsilon + \tau_i\right)^{1/2}}\phi\left(\left(\tilde{a}\left(\pi, A\right)P_R + (1 - \lambda)\mu_i\right)\left(\tau_i + \tau_\varepsilon\right)^{\frac{1}{2}}\right) = c'\left(e_1^*\right), \tag{A.3}$$

where $\phi\left(\cdot\right)$ is the standard normal density function.[34]

The following result summarizes the relationship between the incumbent's level of effort in the first period and the politically relevant parameters:

**Proposition 1.** *(1) The incumbent's level of effort is increasing in the returns to office B. (2) For sufficiently small incumbency advantage (as summarized by $\mu_i$), incumbent's level of effort is decreasing in the level of polarization A, and increasing in the competitiveness of the district (as summarized by the proximity of the normal vote $\pi$ to $\frac{1}{2}$). (3) The incumbent's level of effort is decreasing in incumbency advantage if $\pi > \frac{1}{2}$, and otherwise first increasing, and then decreasing in incumbency advantage.*

**Proof.** The relationship between $e_1^*$ and $B$ is immediate from the inspection of Equation (A.3). The other results are contingent on the proximity of $\tilde{a}\left(\pi, A\right) + (1 - \lambda)\mu_i$ to zero, as $\phi\left(\cdot\right)$ is symmetric and maximized at $\phi(0)$ at which $e^*$ is maximized. If $\mu_i$ is close to zero, then an increase in $A$ or a departure of $\pi$ from $\frac{1}{2}$ in either direction lowers $\phi\left(\left(\tilde{a}\left(\pi, A\right)P_R + (1 - \lambda)\mu_i\right)\left(\tau_i + \tau_\varepsilon\right)^{\frac{1}{2}}\right)$ and hence $e_1^*$.

## A.3 Toward a Defensible Welfare Metric

We define a welfare metric, $w_p$, as the expected utility to a voter from party $p$ stemming from the prospects of electing a copartisan at the end of the first period plus the first-period effort level plus any expected quality advantage that accrues to an incumbent.

To represent the first component, it will be useful to switch from notation denoting the probability a generic incumbent is reelected to that reflecting the probability a generic party $R$ candidate is elected (with its complement denoting the probability a generic party $D$ candidate is elected). The former is given by

$$\Phi\left(\left(\tilde{a}\left(\pi, A\right) + (1 - \lambda)\mu_i P_R\right)\left(\tau_i + \tau_\varepsilon\right)^{\frac{1}{2}}\right)$$

From the foregoing, we have:

$$w_R^* \equiv \Phi\left(\left(\tilde{a}\left(\pi, A\right) + (1 - \lambda)\mu_i P_R\right)\left(\tau_i + \tau_\varepsilon\right)^{\frac{1}{2}}\right)\frac{A}{2} + e_1^*\left(\pi, A, B, \tau_\varepsilon, \tau_i, \mu_i, p_i, 1\right) + (1 - \lambda)\mu_i \mid P_R \mid$$

$$w_D^* \equiv \Phi\left(-\left(\tilde{a}\left(\pi, A\right) + (1 - \lambda)\mu_i P_R\right)\left(\tau_i + \tau_\varepsilon\right)^{\frac{1}{2}}\right)\frac{A}{2} + e_1^*\left(\pi, A, B, \tau_\varepsilon, \tau_i, \mu_i, p_i, 1\right) + (1 - \lambda)\mu_i \mid P_R \mid.$$

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34 Second order conditions for a maximum are established by the convexity of $c\left(\cdot\right)$ and $e_1 - e_1^*$ in equilibrium. While this maximum need not be unique for the general form of $c\left(\cdot\right)$, it is for the specific functional form we adopt below.

To take these measures to data, we first need to impose a functional form on $c(e)$ to derive a closed-form solution for $e_1^*$. We assume $c(e) = \frac{e^2}{2}$, so that the unique value of $e_1^*$ is simply the left side of Equation (A.3). Next, let $\tilde{a}_\circ(\pi) \equiv \frac{\tilde{a}(\pi,A)}{A}$. With this, we can obtain closed form expressions for $w_R^*$ and $w_D^*$ that are functions of $\tilde{a}_\circ(\pi)$ (which is itself a function of election data from the district), and the parameters $A, B, \tau_\varepsilon, \tau_i, \mu_i$. It will be convenient to introduce some changes of variables. Let $\gamma_0 \equiv (1-\lambda)(\tau_\varepsilon + \tau_i)^{1/2}\mu_i$; $\gamma_1 \equiv (\tau_\varepsilon + \tau_i)^{1/2}A$; $\xi \equiv 2\tau_\varepsilon B$. Then

$$w_R^* \equiv \left(\gamma_1 \Phi\left(\gamma_0 P_R + \gamma_1 \tilde{a}_\circ(\pi)\right) + \xi\phi\left(\gamma_0 P_R + \gamma_1 \tilde{a}_\circ(\pi)\right) + 2\gamma_0|P_R|\right)S$$

$$w_D^* \equiv \left(\gamma_1 \Phi\left(-\gamma_0 P_R + \gamma_1 \tilde{a}_\circ(\pi)\right) + \xi\phi\left(\gamma_0 P_R + \gamma_1 \tilde{a}_\circ(\pi)\right) + 2\gamma_0|P_R|\right)S$$

where $S \equiv \frac{1}{2}(\tau_\varepsilon - \tau_i)^{-1/2}$ is simply a normalizing constant. Then for any triple $(\gamma_0, \gamma_1, \xi)$, $\hat{w}_D^*$ and $\hat{w}_R^*$ are identified up to a constant $S$.

Note that one can obtain estimates for $\gamma_0$ and $\gamma_1$ from a simple probit regression of an indicator for whether the Republican candidate wins an election on $\tilde{a}_\circ(\pi)$ (which can itself be derived from a measure of the normal vote); and a dummy variable that takes on a value of 1 if the incumbent is a Republican, $-1$ if a Democrat, and 0 otherwise.

Note also that while $\xi, \gamma_1,$ and $\gamma_0$ are complicated functions of several parameters, each has a substantive interpretation that relates directly to features of a general theory of representation. $\xi$ captures the benefit of holding office, which directly affects the incentive of the incumbent to work hard on behalf of voters. $\gamma_1$ captures polarization, which affects voter welfare in two ways: by potentially muting the effect of district competitiveness on the likelihood that an incumbent retains office and also by raising the value to a typical voter of having a copartisan in office. Lastly, $\gamma_0$ captures incumbency advantage, with larger values corresponding to a greater electoral "edge" for the incumbent party, but also to incumbent quality that is higher in expectation than a randomly drawn challenger, and finally, to related equilibrium effects of both on incumbent effort.

## A.4 Welfare and Disparity Measures

Assuming $K$ equally-sized districts, the aggregate representation-derived welfare of map $\mathcal{M}$ is

$$\hat{W}^*(\mathcal{M}) = \sum_{k=1}^K \left(\pi^k w_R^*\left(\pi_k, p^k; \cdot\right) + \left(1-\pi^k\right)w_D^*\left(\pi^k, p_i^k; \cdot\right)\right)$$

where $\pi^k$ is the normal vote in district $k$ and $p_i^k$ is the incumbent's party in district $k$. A dyadic RD metric (labeled *RD* in the text) of the form $E[w|R] - E[w|D]$ is

$$\hat{\Delta}_w^{RD}(\mathcal{M}) = \frac{\sum_{k=1}^K \pi^k w_R^*\left(\pi^k, p_i^k; \cdot\right)}{\sum_{k=1}^K \pi_k} - \frac{\sum_{k=1}^K \left(1-\pi^k\right)w_D^*\left(\pi^k, p_i^k; \cdot\right)}{\sum_{k=1}^K \left(1-\pi^k\right)}.$$

As noted above, our representational welfare measures are identified up to a constant for any triple $(\gamma_0, \gamma_1, \xi)$. For ease of interpretation we normalize these measures so that they are constrained to lie between zero and 1. This normalization constrains dyadic RD to lie between $-1$ and 1.

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